(16) $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} = \boxed{\sum_{i=1}^{4} \frac{1}{\sqrt{i}}}$

(18) $\frac{3}{5} + \frac{4}{6} + \frac{5}{7} + \frac{6}{8} + \frac{7}{9} = \boxed{\sum_{i=1}^{5} \frac{i+2}{i+4}} = \boxed{\sum_{i=3}^{7} \frac{i}{i+2}}$

(31) a) $\sum_{k=1}^{n} \left[(k+1)^3 - k^3\right] = (\cancel{2^3} - 1^3) + (\cancel{3^3} - \cancel{2^3}) + (4^3 - \cancel{3^3}) + \cdots + ((n+1)^3 - \cancel{n^3})$

$= (n+1)^3 - 1^3 = (n^3 + 3n^2 + 3n + 1) - 1 = \underline{n^3 + 3n^2 + 3n}$

b) $\sum_{k=1}^{n} \left[(k+1)^3 - k^3\right] = \sum_{k=1}^{n} \left[(\cancel{k^3} + 3k^2 + 3k + 1) - \cancel{k^3}\right] = \sum_{k=1}^{n} (3k^2 + 3k + 1)$

$= 3\sum_{k=1}^{n} k^2 + 3\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1 = \underline{3\sum_{k=1}^{n} k^2 + 3 \cdot \frac{n(n+1)}{2} + n}$

c) BY PARTS a) AND b),

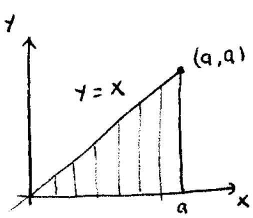
$3\sum_{k=1}^{n} k^2 + 3 \cdot \frac{n(n+1)}{2} + n = n^3 + 3n^2 + 3n$, so

$3\sum_{k=1}^{n} k^2 = n^3 + 3n^2 + 2n - \frac{3n(n+1)}{2} = \frac{2n^3 + 6n^2 + 4n - 3n(n+1)}{2}$

$= \frac{2n^3 + 6n^2 + 4n - 3n^2 - 3n}{2} = \frac{2n^3 + 3n^2 + n}{2} = \frac{n(2n^2 + 3n + 1)}{2} = \frac{n(n+1)(2n+1)}{2}$,

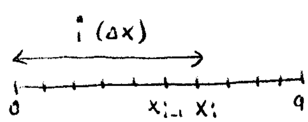
so $\boxed{\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}}$

(38) a)



$\int_0^a x\,dx = A = \frac{1}{2}bh = \boxed{\frac{1}{2}a^2}$

$\underline{\Delta x} = \frac{a-0}{n} = \frac{a}{n}$



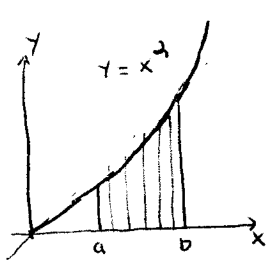
$\underline{c_i} = x_i = 0 + i(\Delta x) = \frac{ai}{n}$

b) $\int_0^a x\,dx = \lim_{\|P\| \to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i$

$= \lim_{\|P\| \to 0} \sum_{i=1}^{n} f\left(\frac{ai}{n}\right) \cdot \frac{a}{n} = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{ai}{n} \cdot \frac{a}{n} = \lim_{n \to \infty} \frac{a^2}{n^2} \sum_{i=1}^{n} i$

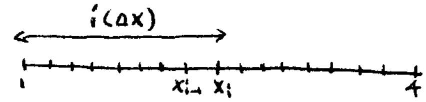
$= \lim_{n \to \infty} \frac{a^2}{n^{\cancel{2}}} \cdot \frac{\cancel{n}(n+1)}{2} = \lim_{n \to \infty} \frac{a^2}{2} \cdot \frac{n+1}{n} = \frac{a^2}{2} \cdot 1 = \boxed{\frac{a^2}{2}}$

(40)



USING EX. 1,

$\int_a^b x^2\,dx = \int_0^b x^2\,dx - \int_0^a x^2\,dx = \frac{b^3}{3} - \frac{a^3}{3} = \boxed{\frac{b^3 - a^3}{3}}$



$\Delta x = \frac{4-1}{n} = \frac{3}{n}$

$c_i = x_i = 1 + i(\Delta x) = 1 + \frac{3i}{n}$

① $$\int_1^4 (x^2+6x)\,dx = \lim_{\|P\|\to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i = \lim_{\|P\|\to 0} \sum_{i=1}^{n} f\left(1+\frac{3i}{n}\right)\cdot\frac{3}{n}$$

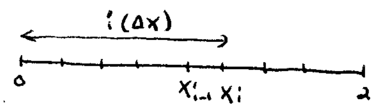
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1+\frac{3i}{n}\right)^2 + 6\left(1+\frac{3i}{n}\right)\right]\cdot\frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[1+\frac{6i}{n}+\frac{9i^2}{n^2}+6+\frac{18i}{n}\right]\cdot\frac{3}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left[7+\frac{24i}{n}+\frac{9i^2}{n^2}\right]\cdot\frac{3}{n}$$

$$= \lim_{n\to\infty} \left[\sum_{i=1}^{n} \frac{21}{n} + \frac{72}{n^2}\sum_{i=1}^{n} i + \frac{27}{n^3}\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \left[n\cdot\frac{21}{n} + \frac{72}{n^2}\cdot\frac{n(n+1)}{2} + \frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty} \left[21 + 36\cdot\frac{n+1}{n} + \frac{9}{2}\cdot\frac{2n^2+3n+1}{n^2}\right] = 21 + 36\cdot 1 + \frac{9}{2}\cdot 2 = \boxed{66}$$



$\Delta x = \frac{2-0}{n} = \frac{2}{n}$

$c_i = x_i = 0 + i(\Delta x) = \frac{2i}{n}$

② $$\int_0^2 (x^3+15x^2)\,dx = \lim_{\|P\|\to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i = \lim_{\|P\|\to 0} \sum_{i=1}^{n} f\left(\frac{2i}{n}\right)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{2i}{n}\right)^3 + 15\left(\frac{2i}{n}\right)^2\right]\cdot\frac{2}{n}$$

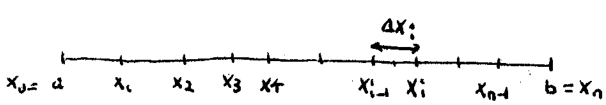
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{8i^3}{n^3}+\frac{60i^2}{n^2}\right]\cdot\frac{2}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{16}{n^4}\cdot i^3 + \frac{120}{n^3}\cdot i^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{16}{n^4}\sum_{i=1}^{n} i^3 + \frac{120}{n^3}\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{16}{n^4}\cdot\frac{n^2(n+1)^2}{4} + \frac{120}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right] = \lim_{n\to\infty} \left[4\cdot\frac{(n+1)^2}{n^2} + 20\cdot\frac{2n^2+3n+1}{n^2}\right]$$

$$= \lim_{n\to\infty} \left[4\cdot\left(\frac{n+1}{n}\right)^2 + 20\cdot\frac{2n^2+3n+1}{n^2}\right] = 4\cdot 1 + 20\cdot 2 = \boxed{44}$$

③



$c_i = \sqrt{x_{i-1}\,x_i}$ FOR $1 \le i \le n$

$$\int_a^b \frac{1}{x^2}\,dx = \lim_{\|P\|\to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i = \lim_{\|P\|\to 0} \sum_{i=1}^{n} \frac{1}{c_i^2}(x_i - x_{i-1})$$

$$= \lim_{\|P\|\to 0} \sum_{i=1}^{n} \frac{1}{x_{i-1}x_i}(x_i - x_{i-1}) = \lim_{\|P\|\to 0} \sum_{i=1}^{n} \left(\frac{1}{x_{i-1}} - \frac{1}{x_i}\right)$$

$$= \lim_{\|P\|\to 0} \left[\left(\frac{1}{x_0} - \cancel{\frac{1}{x_1}}\right) + \left(\cancel{\frac{1}{x_1}} - \cancel{\frac{1}{x_2}}\right) + \left(\cancel{\frac{1}{x_2}} - \cancel{\frac{1}{x_3}}\right) + \cdots + \left(\cancel{\frac{1}{x_{n-1}}} - \frac{1}{x_n}\right)\right]$$

$$= \lim_{\|P\|\to 0} \left[\frac{1}{x_0} - \frac{1}{x_n}\right] = \lim_{\|P\|\to 0} \left[\frac{1}{a} - \frac{1}{b}\right] = \boxed{\frac{1}{a} - \frac{1}{b}}$$