(11) $\frac{dY}{dX} = 3Y$, $Y_0 = 2$ WHEN $X_0 = 0$

$\int \frac{1}{Y}\,dY = \int 3\,dX$, $\ln Y = 3X + C$, $\underline{Y} = e^{3X+C} = e^C \cdot e^{3X} = \underline{Ae^{3X}}$

WHEN $X = 0$, $Y = 2$; SO $2 = A \cdot 1$ AND $\underline{A = 2}$: $\boxed{Y = 2e^{3X}}$

REMARK SINCE THIS IS THE DE FOR EXPONENTIAL GROWTH, WE COULD ALSO GET THIS ANSWER FROM THE EXPONENTIAL GROWTH FORMULA.

(15) $\frac{dh}{ds} = 2h + 1$, $h(0) = 4$

$\int \frac{1}{2h+1}\,dh = \int ds$, $\frac{1}{2}\int \frac{2}{2h+1}\,dh = s + C$, $\frac{1}{2}\ln(2h+1) = s + C$, $\ln(2h+1) = 2s + D$

$2h+1 = e^{2s+D} = e^D e^{2s} = Ae^{2s}$, $2h = Ae^{2s} - 1$, $\underline{h = \frac{1}{2}(Ae^{2s} - 1)}$

WHEN $s = 0$, $h = 4$: $4 = \frac{1}{2}(A \cdot 1 - 1)$ SO $\underline{A = 9}$: $\boxed{h = \frac{1}{2}(9e^{2s} - 1)}$

(16) $\frac{dN}{dT} = 5 - N$, $N(2) = 3$

$\int \frac{1}{5-N}\,dN = \int dT$, $-\int \frac{-1}{5-N}\,dN = T + C$, $-\ln(5-N) = T + C$, $\ln(5-N) = -T - C$,

$5 - N = e^{-T-C} = e^{-C}e^{-T} = Ae^{-T}$, $\underline{N = 5 - Ae^{-T}}$

WHEN $T = 2$, $N = 3$; SO $3 = 5 - Ae^{-2}$, $Ae^{-2} = 2$, $\underline{A = 2e^2}$

SO $\boxed{N = 5 - (2e^2)e^{-T} = 5 - 2e^{2-T}}$

(17) $\frac{dN}{dT} = .3\,N(T)$, $N(0) = 20$

$\int \frac{1}{N}\,dN = \int .3\,dT$, $\ln N = .3T + C$, $\underline{N} = e^{.3T+C} = e^C e^{.3T} = \underline{Ae^{.3T}}$

WHEN $T = 0$, $N = 20$; SO $\underline{20} = A \cdot 1 = \underline{A}$ AND $\boxed{N = 20e^{.3T}}$

REMARK WE COULD ALSO GET THIS ANSWER FROM THE EXPONENTIAL GROWTH FORMULA.

$N(5) = \boxed{20e^{1.5}} \approx 90$

(19) a) $\frac{1}{N}\cdot\frac{dN}{dT} = r$ SO $\int \frac{1}{N}\,dN = \int r\,dT$, $\ln N = rT + C$, $\underline{N} = e^{rT+C} = e^C e^{rT} = \underline{Ae^{rT}}$

WHERE $T = 0$ GIVES $N(0) = A \cdot 1 = A$, SO $\boxed{N = N_0 e^{rT}}$

b) TAKING LOGARITHMS GIVES $\log N = \log N_0 + (rT)\log e$ OR $\underline{\log N = \log N_0 + (r\log e)T}$

IF WE GRAPH N AS A FUNCTION OF T ON SEMILOG PAPER,

$r = \frac{m}{\log e}$ WHERE m IS THE SLOPE OF THE LINE.

c) PLOT THE DATA ON SEMILOG PAPER, FIND THE SLOPE m OF THE RESULTING LINE, AND THEN USE $r = \frac{m}{\log e}$ (WHERE $\log e \approx .434$)

(20) a) $\frac{dW}{dT} = -\lambda W$, $W(0) = W_0$ SO $\boxed{W = W_0 e^{-\lambda T}}$ BY THE EXPONENTIAL GROWTH/DECAY FORMULA.

b) $\underline{W = 123e^{-\lambda T}}$ SINCE $W_0 = 123$, AND $W(5) = 20$ GIVES $123e^{-5\lambda} = 20$, $e^{-5\lambda} = \frac{20}{123}$,

$-5\lambda = \ln\frac{20}{123}$, $\lambda = \boxed{-\frac{1}{5}\ln\frac{20}{123}} = \boxed{\frac{1}{5}\ln\frac{123}{20}}$ (SINCE $\underline{\ln\frac{1}{T} = -\ln T}$)

WHEN $W = \frac{1}{2}(123)$, $\cancel{123}e^{-\lambda T} = \frac{1}{2}(\cancel{123})$, $e^{-\lambda T} = \frac{1}{2}$, $-\lambda T = \ln\frac{1}{2}$, $T = \frac{\ln\frac{1}{2}}{-\lambda}$

SO $T = \boxed{(5\ln\tfrac{1}{2})/(\ln\tfrac{20}{123})\text{ MIN}} = \boxed{5\left(\frac{\ln 2}{\ln\frac{123}{20}}\right)\text{ MIN}} \approx 1.9$ MIN

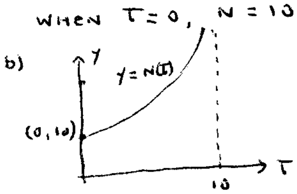
MATH 17B

SEC. 8.1 – SOLUTIONS

21 $\frac{dN}{dT} = \frac{1}{100}N^2$, $N(0) = 10$

a) $\int \frac{1}{N^2}\,dN = \int \frac{1}{100}\,dT$, $-\frac{1}{N} = \frac{1}{100}T + C$, $\frac{1}{N} = D - \frac{T}{100}$, $N = \frac{1}{D - \frac{T}{100}} = \frac{100}{E - T}$

WHEN $T = 0$, $N = 10$; SO $10 = \frac{100}{E}$, $E = 10$, AND $\boxed{N = \frac{100}{10 - T}}$

b) 

AS $T \to 10^-$, $N(T) \to \infty$; SO THE POPULATION GROWS WITHOUT BOUND AS T APPROACHES 10 (FROM THE LEFT)

22 $\frac{dL}{dT} = K(34 - L)$, $L(0) = 2$

a) $(-)\int \frac{(-1)}{34 - L}\,dL = \int K\,dT$, $-\ln(34 - L) = KT + C$, $\ln(34 - L) = -KT - C$,

$34 - L = e^{-KT-C} = e^{-C}e^{-KT} = Ae^{-KT}$, $L = 34 - Ae^{-KT}$

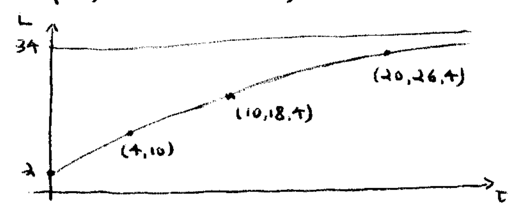
WHEN $T = 0$, $L = 2$; SO $2 = 34 - A \cdot 1$, $A = 32$, AND $\boxed{L = 34 - 32e^{-KT}}$

b) $L(4) = 10$, SO $10 = 34 - 32e^{-4K}$, $32e^{-4K} = 24$, $e^{-4K} = \frac{3}{4}$, $-4K = \ln\frac{3}{4}$,

$K = \boxed{-\frac{1}{4}\ln\frac{3}{4}} = \boxed{\frac{1}{4}\ln\frac{4}{3}} \approx .072$

c) $L(10) = 34 - 32\left(\frac{3}{4}\right)^{10/4} = \boxed{34 - 32\left(\frac{3}{4}\right)^{5/2}} \approx 18.4$

d) $L_\infty = \lim_{T\to\infty} L(T) = \lim_{T\to\infty}\left(34 - \frac{32}{e^{KT}}\right) = 34 - 0 = \boxed{34}$



25 $\frac{dY}{dX} = Y(1 + Y)$, $Y = 2$ WHEN $X = 0$

$\int \frac{1}{Y(1+Y)}\,dY = \int dX$, $\int \left(\frac{1}{Y} - \frac{1}{1+Y}\right) dY = X + C$,

$\frac{1}{Y(1+Y)} = \frac{A}{Y} + \frac{B}{1+Y}$

$1 = A(1+Y) + BY$

$Y = 0$: $1 = A$

$Y = -1$: $1 = -B$ SO $B = -1$

$\ln Y - \ln(1+Y) = X + C$, $\ln\left(\frac{Y}{1+Y}\right) = X + C$,

$\frac{Y}{1+Y} = e^{X+C} = e^C e^X = Ae^X$, $\frac{1+Y}{Y} = \frac{1}{Ae^X}$,

$\frac{1}{Y} + 1 = ae^{-X}$, $\frac{1}{Y} = ae^{-X} - 1$, $Y = \frac{1}{ae^{-X} - 1}$

WHEN $X = 0$, $Y = 2$: $2 = \frac{1}{a - 1}$, $a - 1 = \frac{1}{2}$, $a = \frac{3}{2}$ SO $\boxed{Y = \frac{1}{\frac{3}{2}e^{-X} - 1} = \frac{2}{3e^{-X} - 2}}$

32 $\frac{dY}{dX} = (1 + Y)^2$

$u = 1 + Y$, $du = dY$

$\int \frac{1}{(1+Y)^2}\,dY = \int dX$, $\int (1+Y)^{-2}\,dY = X + C$, $\int u^{-2}\,du = X + C$, $-\frac{1}{u} = X + C$,

$-\frac{1}{1+Y} = X + C$, $\frac{1}{1+Y} = D - X$, $1 + Y = \frac{1}{D - X}$, $\boxed{Y = \frac{1}{D - X} - 1}$

36 $\frac{dY}{dX} = Y^2 + 4$, PASSES THROUGH $(0, 2)$

$\int \frac{1}{Y^2 + 4}\,dY = \int dX$, $\frac{1}{2}\tan^{-1}\frac{Y}{2} = X + C$, $\tan^{-1}\frac{Y}{2} = 2X + D$,

WHEN $X = 0$, $Y = 2$; SO $\tan^{-1} 1 = D$ AND $D = \frac{\pi}{4}$: $\tan^{-1}\frac{Y}{2} = 2X + \frac{\pi}{4}$

SO $\tan\left(\tan^{-1}\frac{Y}{2}\right) = \tan\left(2X + \frac{\pi}{4}\right)$,

$\frac{Y}{2} = \tan\left(2X + \frac{\pi}{4}\right)$, $\boxed{Y = 2\tan\left(2X + \frac{\pi}{4}\right)}$