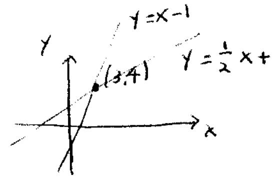
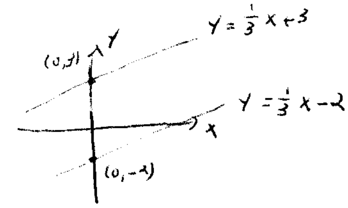
MATH 178

SOLUTIONS – Sec. 9.1





①
$$\begin{array}{rl} & x - y = 1 \\ \ominus & x - 2y = -2 \\ \hline & \boxed{y = 3} \end{array} \qquad x = y + 1, \text{ so } \boxed{x = 4}$$

③
$$\begin{array}{l} x - 3y = 6 \\ y = 3 + \frac{1}{3}x \end{array} \qquad x - 3(3 + \tfrac{1}{3}x) = 6 \qquad -9 = 6 \qquad \boxed{\text{NO SOLUTION}}$$

⑤
$$\begin{array}{l} 2x - 3y = 5 \\ 4x - 6y = c \end{array} \qquad \begin{array}{rl} & 4x - 6y = 10 \\ - & 4x - 6y = c \\ \hline & 0 = 10 - c \end{array}$$

a) IF $\underline{c = 10}$, THERE ARE INFINITELY MANY SOLUTIONS (SINCE THE LINES ARE IDENTICAL).

b) IF $\underline{c \neq 10}$, THERE IS NO SOLUTION (SINCE THE LINES ARE PARALLEL).

c) SINCE THE LINES ARE IDENTICAL OR PARALLEL, THEY CANNOT INTERSECT IN A SINGLE POINT.

⑰
$$\begin{array}{l} x + y = 11 \\ 2.3x + 1.7y = 21.7 \end{array} \qquad \begin{array}{rl} & 23x + 23y = 253 \\ \ominus & 23x + 17y = 217 \\ \hline & 6y = 36 \end{array} \qquad \boxed{y = 6} \quad \text{AND} \quad x = 11 - y \text{ so } \boxed{x = 5}$$

㉒
$$\begin{bmatrix} 1 & 4 & -3 & -13 \\ 2 & -3 & 5 & 18 \\ 3 & 1 & -2 & 1 \end{bmatrix} \xrightarrow[-3R_1+R_3]{-2R_1+R_2} \begin{bmatrix} 1 & 4 & -3 & -13 \\ 0 & -11 & 11 & 44 \\ 0 & -11 & 7 & 40 \end{bmatrix} \xrightarrow[-\frac{1}{11}R_2]{-R_2+R_3} \begin{bmatrix} 1 & 4 & -3 & -13 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & -4 & -4 \end{bmatrix} \xrightarrow[-\frac{1}{4}R_3]{-4R_2+R_1} \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\xrightarrow[R_3+R_2]{-R_3+R_1} \begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} \qquad \boxed{\begin{array}{l} x = 2 \\ y = -3 \\ z = 1 \end{array}}$$

㉓
$$\begin{bmatrix} -2 & 4 & -1 & -1 \\ 1 & 7 & 2 & -4 \\ 3 & -2 & 3 & -3 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 7 & 2 & -4 \\ -2 & 4 & -1 & -1 \\ 3 & -2 & 3 & -3 \end{bmatrix} \xrightarrow[-3R_1+R_3]{2R_1+R_2} \begin{bmatrix} 1 & 7 & 2 & -4 \\ 0 & 18 & 3 & -9 \\ 0 & -23 & -3 & 9 \end{bmatrix} \xrightarrow{R_2+R_3} \begin{bmatrix} 1 & 7 & 2 & -4 \\ 0 & 18 & 3 & -9 \\ 0 & -5 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{5}R_3} \begin{bmatrix} 1 & 7 & 2 & -4 \\ 0 & 18 & 3 & -9 \\ 0 & 1 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 7 & 2 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 18 & 3 & -9 \end{bmatrix} \xrightarrow[-18R_2+R_3]{-7R_2+R_1} \begin{bmatrix} 1 & 0 & 2 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & -9 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{3}R_3} \begin{bmatrix} 1 & 0 & 2 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \end{bmatrix} \xrightarrow{-2R_3+R_1} \begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \end{bmatrix} \end{array} \qquad \boxed{\begin{array}{l} x = 2 \\ y = 0 \\ z = -3 \end{array}}$$

㉔
$$\begin{bmatrix} 2 & -1 & 3 & 3 \\ 2 & 1 & 4 & 4 \\ 2 & -3 & 2 & 2 \end{bmatrix} \xrightarrow[\substack{-R_1+R_3 \\ \frac{1}{2}R_1}]{-R_1+R_2} \begin{bmatrix} 1 & -1/2 & 3/2 & 3/2 \\ 0 & 2 & 1 & 1 \\ 0 & -2 & -1 & -1 \end{bmatrix} \xrightarrow[\frac{1}{2}R_2]{R_2+R_3} \begin{bmatrix} 1 & -1/2 & 3/2 & 3/2 \\ 0 & 1 & 1/2 & 1/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}R_2+R_1} \begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 0 & 7/4 & 7/4 \\ 0 & 1 & 1/2 & 1/2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{l} x + \frac{7}{4}z = \frac{7}{4} \\ y + \frac{1}{2}z = \frac{1}{2} \end{array} \qquad \text{LET } \underline{z = 4t}: \quad \boxed{\begin{array}{l} x = \frac{7}{4} - 7t \\ y = \frac{1}{2} - 2t \\ z = 4t \end{array}}, \ t \text{ in } \mathbb{R}$$

REMARK IF WE LET $\underline{z = t}$ INSTEAD, WE GET THE SOLUTION $\boxed{x = \frac{7}{4} - \frac{7}{4}t, \quad y = \frac{1}{2} - \frac{1}{2}t, \quad z = t}$, t IN $\mathbb{R}$

㉕
$$\begin{bmatrix} -1 & -2 & 3 & -9 \\ 2 & 1 & -1 & 5 \\ 4 & -3 & 5 & -9 \end{bmatrix} \xrightarrow[4R_1+R_3]{2R_1+R_2} \begin{bmatrix} 1 & 2 & -3 & 9 \\ 0 & -3 & 5 & -13 \\ 0 & -11 & 17 & -45 \end{bmatrix} \xrightarrow[R_2 \leftrightarrow R_3]{-4R_2+R_3} \begin{bmatrix} 1 & 2 & -3 & 9 \\ 0 & 1 & -3 & 7 \\ 0 & -3 & 5 & -13 \end{bmatrix} \xrightarrow[3R_2+R_3]{-2R_2+R_1} \begin{bmatrix} 1 & 0 & 3 & -5 \\ 0 & 1 & -3 & 7 \\ 0 & 0 & -4 & 8 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{4}R_3} \begin{bmatrix} 1 & 0 & 3 & -5 \\ 0 & 1 & -3 & 7 \\ 0 & 0 & 1 & -2 \end{bmatrix} \xrightarrow[-3R_3+R_1]{3R_3+R_2} \begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \end{array} \qquad \boxed{\begin{array}{l} x = 1 \\ y = 1 \\ z = -2 \end{array}}$$

MATH 17B

SOLUTIONS - Sec. 9.1

(29)

$$\begin{bmatrix} 1 & -2 & 1 & 3 \\ 2 & -3 & 1 & 8 \end{bmatrix} \xrightarrow{-2R_1+R_2} \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 1 & -1 & 2 \end{bmatrix} \xrightarrow{2R_2+R_1} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 7 \\ 0 & 1 & -1 & 2 \end{bmatrix} \end{array}$$

$x - z = 7$

$y - z = 2$

Let $z = t$: $\boxed{x = 7 + t, \quad y = 2 + t, \quad z = t}$, t in $\mathbb{R}$

(30)

$$\begin{bmatrix} 1 & -1 & 0 & 2 \\ 1 & 1 & 1 & 3 \end{bmatrix} \xrightarrow{-R_1+R_2} \begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 2 & 1 & 1 \end{bmatrix} \xrightarrow{\frac{1}{2}R_2} \begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 1/2 & 1/2 \end{bmatrix} \xrightarrow{R_2+R_1}$$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 1 & 0 & 1/2 & 5/2 \\ 0 & 1 & 1/2 & 1/2 \end{bmatrix} \end{array}$$

$x + \frac{1}{2}z = 5/2$

$y + \frac{1}{2}z = 1/2$

Let $z = 2t$: $\boxed{\begin{aligned} x &= \tfrac{5}{2} - t \\ y &= \tfrac{1}{2} - t \\ z &= 2t \end{aligned}}$, t in $\mathbb{R}$

REMARK: If we let $z = t$ instead, we get the solution $\boxed{x = \frac{5}{2} - \frac{1}{2}t, \quad y = \frac{1}{2} - \frac{1}{2}t, \quad z = t}$, t in $\mathbb{R}$

(35)

$.24x + .21y + .17z = 500$

$.04x + .07y = 100$

$.08x + .12y = 180$

$24x + 21y + 17z = 50{,}000$

$4x + 7y = 10{,}000$

$8x + 12y = 18{,}000$

$$\begin{bmatrix} 24 & 21 & 17 & 50{,}000 \\ 4 & 7 & 0 & 10{,}000 \\ 8 & 12 & 0 & 18{,}000 \end{bmatrix} \xrightarrow[R_3 \leftrightarrow R_1]{\frac{1}{8}R_3} \begin{bmatrix} 1 & 3/2 & 0 & 2{,}250 \\ 4 & 7 & 0 & 10{,}000 \\ 24 & 21 & 17 & 50{,}000 \end{bmatrix} \xrightarrow[-24R_1+R_3]{-4R_1+R_2} \begin{bmatrix} 1 & 3/2 & 0 & 2250 \\ 0 & 1 & 0 & 1000 \\ 0 & -15 & 17 & -4000 \end{bmatrix}$$

$$\xrightarrow[15R_2+R_3]{-\frac{3}{2}R_2+R_1} \begin{bmatrix} 1 & 0 & 0 & 750 \\ 0 & 1 & 0 & 1000 \\ 0 & 0 & 17 & 11000 \end{bmatrix} \xrightarrow{\frac{1}{17}R_3} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 750 \\ 0 & 1 & 0 & 1000 \\ 0 & 0 & 1 & 11000/17 \end{bmatrix} \end{array}$$

$$\boxed{\begin{array}{l} 750 \text{ g of Type 1} \\ 1000 \text{ g of Type 2} \\ \frac{11{,}000}{17} \text{ g of Type 3} \end{array}}$$