MATH 17B SOLUTIONS – Sec. 9.2

(3) $A+B = 2A - B + D$, so $D = 2B - A = \begin{bmatrix} 0 & 2 \\ 4 & 8 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 \\ 4 & 11 \end{bmatrix}}$

(15) $A = \begin{bmatrix} -1 & 0 & 3 \\ 2 & 1 & -4 \end{bmatrix}$, so $A^T = \boxed{\begin{bmatrix} -1 & 2 \\ 0 & 1 \\ 3 & -4 \end{bmatrix}}$

(21) a) $AB = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} -2 & -3 \\ 0 & 5 \end{bmatrix}}$

b) $BA = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 6 \\ 2 & 2 \end{bmatrix}}$

(28) $A: 3\times 4$ $B: m \times n$

a) AB is defined when $\underline{m=4}$

b) BA is defined when $\underline{n=3}$

(31) a) $AB = \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 1 & 3 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 7 & 5 & 9 & -1 \\ -4 & -2 & -6 & 0 \end{bmatrix}}$

b) $B^T A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & -1 \\ 2 & 4 \\ 0 & -6 \\ -1 & -3 \end{bmatrix}}$

(33) $A = \begin{bmatrix} 2 & 1 \\ -1 & -3 \end{bmatrix}$

$\underline{A^2} = \begin{bmatrix} 2 & 1 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -3 \end{bmatrix} = \boxed{\begin{bmatrix} 3 & -1 \\ 1 & 8 \end{bmatrix}}$

$\underline{A^3} = AA^2 = \begin{bmatrix} 2 & 1 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 1 & 8 \end{bmatrix} = \boxed{\begin{bmatrix} 7 & 6 \\ -6 & -23 \end{bmatrix}}$

$\underline{A^4} = AA^3 = \begin{bmatrix} 2 & 1 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} 7 & 6 \\ -6 & -23 \end{bmatrix} = \boxed{\begin{bmatrix} 8 & -11 \\ 11 & 63 \end{bmatrix}}$

(39)
$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 0 \\ 2x_2 + x_3 &= 1 \\ x_1 \qquad - 2x_3 &= 2 \end{aligned}$$

$\underline{Ax = b}$ where $A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 2 & 1 \\ 1 & 0 & -2 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$.

(42)
$$\begin{aligned} x_1 - 2x_2 + x_3 &= 1 \\ -2x_1 + x_2 - 3x_3 &= 0 \end{aligned}$$

$\underline{Ax = b}$ where $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 1 & -3 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(44) If $A = \begin{bmatrix} 2 & 3 & 1 \\ 5 & 2 & 3 \\ 1 & 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -6/5 & 2/5 & 7/5 \\ 3/5 & -1/5 & -1/5 \\ 8/5 & -1/5 & -11/5 \end{bmatrix}$,

$\underline{AB} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \underline{I}$ and $\underline{BA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \underline{I}$, so $B = A^{-1}$!

<u>REMARK</u> It is actually sufficient to check one of these two conditions; if either $AB = I$ or $BA = I$, then $B = A^{-1}$!

(45) $A = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$ so $A^{-1} = \frac{1}{-5}\begin{bmatrix} 3 & -1 \\ -2 & -1 \end{bmatrix} = \boxed{\begin{bmatrix} -3/5 & 1/5 \\ 2/5 & 1/5 \end{bmatrix}}$

(46) $B = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$ so $B^{-1} = \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} 1/5 & 1/5 \\ -3/10 & 1/5 \end{bmatrix}}$

REMARK WE CAN CHECK THE ANSWERS TO #45 AND #46 BY CHECKING THAT $AA^{-1} = I$ AND $BB^{-1} = I$.

(51) $A = \begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix}$, $D = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$

a) $AX = D$ GIVES $\begin{bmatrix} -1 & 0 & -2 \\ 2 & -3 & -5 \end{bmatrix} \xrightarrow[-R_1]{2R_1+R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & -3 & -9 \end{bmatrix} \xrightarrow{-\frac{1}{3}R_2} \begin{array}{c} \begin{matrix} x & y & \end{matrix} \\ \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix} \end{array}$ $\boxed{\begin{matrix} x_1 = 2 \\ x_2 = 3 \end{matrix}}$

SO $X = \boxed{\begin{bmatrix} 2 \\ 3 \end{bmatrix}}$

b) SINCE $A^{-1} = \frac{1}{3}\begin{bmatrix} -3 & 0 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -2/3 & -1/3 \end{bmatrix}$,

$AX = D$ GIVES $X = A^{-1}D = \begin{bmatrix} -1 & 0 \\ -2/3 & -1/3 \end{bmatrix}\begin{bmatrix} -2 \\ -5 \end{bmatrix} = \boxed{\begin{bmatrix} 2 \\ 3 \end{bmatrix}}$

(52) a) IF $X = AX + D$, THEN $IX = AX + D \implies IX - AX = D$

SO $(I - A)X = D$. IF $I - A$ IS INVERTIBLE, THEN

$(I-A)^{-1}\left[(I-A)X\right] = (I-A)^{-1}D$ SO $IX = (I-A)^{-1}D$ AND $X = (I-A)^{-1}D$.

b) $A = \begin{bmatrix} 3 & 2 \\ 0 & -1 \end{bmatrix}$, $D = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 0 & 2 \end{bmatrix}$,

SO $(I-A)^{-1} = \frac{1}{-4}\begin{bmatrix} 2 & 2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -1/2 & -1/2 \\ 0 & 1/2 \end{bmatrix}$

AND $X = (I-A)^{-1}D = \begin{bmatrix} -1/2 & -1/2 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} -2 \\ 2 \end{bmatrix} = \boxed{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}$