SOLUTIONS – SEC. 9.2

(51) $AX = D$ WHERE $A = \begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix}$ AND $D = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$

a) $\begin{bmatrix} -1 & 0 & -2 \\ 2 & -3 & -5 \end{bmatrix} \xrightarrow[-R_1]{2R_1+R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & -3 & -9 \end{bmatrix} \xrightarrow{-\frac{1}{3}R_2} \begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix} \end{array}$ $\quad \underline{x = 2}$, $\underline{y = 3}$ SO $X = \begin{bmatrix} x \\ y \end{bmatrix} = \boxed{\begin{bmatrix} 2 \\ 3 \end{bmatrix}}$

b) $A^{-1} = \frac{1}{3}\begin{bmatrix} -3 & 0 \\ -2 & -1 \end{bmatrix}$, SO $\underline{X = A^{-1}D} = \frac{1}{3}\begin{bmatrix} -3 & 0 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} -2 \\ -5 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 6 \\ 9 \end{bmatrix} = \boxed{\begin{bmatrix} 2 \\ 3 \end{bmatrix}}$

(58) $A = \begin{bmatrix} a & 8 \\ 2 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

a) IF $a \neq 4$, $\det(A) = 4a - 16 \neq 0$; SO A IS INVERTIBLE AND THEREFORE $AX = B$ HAS THE UNIQUE SOLUTION $X = A^{-1}B$.

b) IF $a = 4$, REDUCING THE AUGMENTED MATRIX GIVES

$$\begin{bmatrix} 4 & 8 & b_1 \\ 2 & 4 & b_2 \end{bmatrix} \xrightarrow[\frac{1}{2}R_2]{\frac{1}{4}R_1} \begin{bmatrix} 1 & 2 & b_1/4 \\ 1 & 2 & b_2/2 \end{bmatrix} \xrightarrow{-R_1+R_2} \begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \begin{bmatrix} 1 & 2 & \frac{1}{4}b_1 \\ 0 & 0 & \frac{1}{4}(2b_2 - b_1) \end{bmatrix} \end{array}$$

i) IF $\underline{b_1 = 2b_2}$, THERE ARE INFINITELY MANY SOLUTIONS (SINCE y IS A FREE VARIABLE).

ii) IF $\underline{b_1 \neq 2b_2}$, THERE IS NO SOLUTION SINCE $\frac{1}{4}(2b_2 - b_1) \neq 0$.

c) 1) IF $a \neq 4$, THE LINES $ax + 8y = b_1$ AND $2x + 4y = b_2$ ARE <u>NOT PARALLEL</u>, SO THEY INTERSECT AT ONE POINT.

2) IF $a = 4$, THE LINES $4x + 8y = b_1$ AND $2x + 4y = b_2$ ARE

i) <u>IDENTICAL</u> IF $b_1 = 2b_2$, SO THERE ARE INFINITELY MANY PTS. OF INTERSECTION.

ii) <u>PARALLEL</u> IF $b_1 \neq 2b_2$, SO THERE ARE NO POINTS OF INTERSECTION.

(67) $A = \begin{bmatrix} 2 & -1 & -1 \\ 2 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 2 & -1 & -1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ -1 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-R_1]{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & -1 & 1 & 0 & 0 & -1 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 2 & -1 & -1 & 1 & 0 & 0 \end{bmatrix} \xrightarrow[-2R_1+R_3]{-2R_1+R_2} \begin{bmatrix} 1 & -1 & 1 & 0 & 0 & -1 \\ 0 & 3 & -1 & 0 & 1 & 2 \\ 0 & 1 & -3 & 1 & 0 & 2 \end{bmatrix} \xrightarrow[\substack{R_2+R_1 \\ -3R_2+R_3}]{R_2 \leftrightarrow R_3}$$

$$\begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 1 \\ 0 & 1 & -3 & 1 & 0 & 2 \\ 0 & 0 & 8 & -3 & 1 & -4 \end{bmatrix} \xrightarrow{\frac{1}{8}R_3} \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 1 \\ 0 & 1 & -3 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3/8 & 1/8 & -1/2 \end{bmatrix} \xrightarrow[3R_3+R_2]{2R_3+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/4 & 1/4 & 0 \\ 0 & 1 & 0 & -1/8 & 3/8 & 1/2 \\ 0 & 0 & 1 & -3/8 & 1/8 & -1/2 \end{array}\right]$$

THEREFORE $\boxed{A^{-1} = \begin{bmatrix} 1/4 & 1/4 & 0 \\ -1/8 & 3/8 & 1/2 \\ -3/8 & 1/8 & -1/2 \end{bmatrix}}$

<u>REMARK</u> NOTICE THAT WE CAN CHECK THIS ANSWER BY SEEING IF $\underline{AA^{-1} = I}$ OR IF $\underline{A^{-1}A = I}$.

MATH 17B SOLUTIONS – SEC. 9.2

(68) $A = \begin{bmatrix} -1 & 3 & -1 \\ 2 & -2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} -1 & 3 & -1 & 1 & 0 & 0 \\ 2 & -2 & 3 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\substack{-R_1+R_3 \\ -R_1}]{2R_1+R_2} \begin{bmatrix} 1 & -3 & 1 & -1 & 0 & 0 \\ 0 & 4 & 1 & 2 & 1 & 0 \\ 0 & -2 & 3 & -1 & 0 & 1 \end{bmatrix} \xrightarrow{\frac{1}{4}R_2} \begin{bmatrix} 1 & -3 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1/4 & 1/2 & 1/4 & 0 \\ 0 & -2 & 3 & -1 & 0 & 1 \end{bmatrix} \xrightarrow[2R_2+R_3]{3R_2+R_1}$$

$$\begin{bmatrix} 1 & 0 & 7/4 & 1/2 & 3/4 & 0 \\ 0 & 1 & 1/4 & 1/2 & 1/4 & 0 \\ 0 & 0 & 7/2 & 0 & 1/2 & 1 \end{bmatrix} \xrightarrow{\frac{2}{7}R_3} \begin{bmatrix} 1 & 0 & 7/4 & 1/2 & 3/4 & 0 \\ 0 & 1 & 1/4 & 1/2 & 1/4 & 0 \\ 0 & 0 & 1 & 0 & 1/7 & 2/7 \end{bmatrix} \xrightarrow[-\frac{1}{4}R_3+R_2]{-\frac{7}{4}R_3+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 1 & 0 & 1/2 & 3/14 & -1/14 \\ 0 & 0 & 1 & 0 & 1/7 & 2/7 \end{array}\right]$$

THEREFORE $\boxed{A^{-1} = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ 1/2 & 3/14 & -1/14 \\ 0 & 1/7 & 2/7 \end{bmatrix}}$

(70) $A = \begin{bmatrix} -1 & 0 & 2 \\ -1 & -2 & 3 \\ 0 & 2 & -1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} -1 & 0 & 2 & 1 & 0 & 0 \\ -1 & -2 & 3 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-R_1]{-R_1+R_2} \begin{bmatrix} 1 & 0 & -2 & -1 & 0 & 0 \\ 0 & -2 & 1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & -1 & 0 & 0 \\ 0 & -2 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{array}\right]$$

A IS <u>NOT INVERTIBLE</u>, SINCE THE LEFT HALF OF THE LAST ROW IS ALL ZEROS.

(71) $L = \begin{bmatrix} 0 & 3.2 & 1.7 \\ .2 & 0 & 0 \\ 0 & .7 & 0 \end{bmatrix}$ $\qquad N(0) = \begin{bmatrix} 2000 \\ 800 \\ 200 \end{bmatrix}$

$$N(1) = LN(0) = \begin{bmatrix} 0 & 3.2 & 1.7 \\ .2 & 0 & 0 \\ 0 & .7 & 0 \end{bmatrix} \begin{bmatrix} 2000 \\ 800 \\ 200 \end{bmatrix} = \begin{bmatrix} 2900 \\ 400 \\ 560 \end{bmatrix}$$

$$N(2) = LN(1) = \begin{bmatrix} 0 & 3.2 & 1.7 \\ .2 & 0 & 0 \\ 0 & .7 & 0 \end{bmatrix} \begin{bmatrix} 2900 \\ 400 \\ 560 \end{bmatrix} = \boxed{\begin{bmatrix} 2232 \\ 580 \\ 280 \end{bmatrix}}$$

(72) $L = \begin{bmatrix} 0 & 1.6 & 3.9 \\ .8 & 0 & 0 \\ 0 & .1 & 0 \end{bmatrix}$ $\qquad N(0) = \begin{bmatrix} 1000 \\ 100 \\ 20 \end{bmatrix}$

$$N(1) = LN(0) = \begin{bmatrix} 0 & 1.6 & 3.9 \\ .8 & 0 & 0 \\ 0 & .1 & 0 \end{bmatrix} \begin{bmatrix} 1000 \\ 100 \\ 20 \end{bmatrix} = \begin{bmatrix} 238 \\ 800 \\ 10 \end{bmatrix}$$

$$N(2) = LN(1) = \begin{bmatrix} 0 & 1.6 & 3.9 \\ .8 & 0 & 0 \\ 0 & .1 & 0 \end{bmatrix} \begin{bmatrix} 238 \\ 800 \\ 10 \end{bmatrix} = \begin{bmatrix} 1319 \\ 190.4 \\ 80 \end{bmatrix} \approx \boxed{\begin{bmatrix} 1319 \\ 190 \\ 80 \end{bmatrix}}$$

(73) $L = \begin{bmatrix} 0 & 0 & 4.6 & 3.7 \\ .7 & 0 & 0 & 0 \\ 0 & .5 & 0 & 0 \\ 0 & 0 & .1 & 0 \end{bmatrix}$

$$N(1) = LN(0) = \begin{bmatrix} 0 & 0 & 4.6 & 3.7 \\ .7 & 0 & 0 & 0 \\ 0 & .5 & 0 & 0 \\ 0 & 0 & .1 & 0 \end{bmatrix} \begin{bmatrix} 1500 \\ 500 \\ 250 \\ 50 \end{bmatrix} = \begin{bmatrix} 1335 \\ 1050 \\ 250 \\ 25 \end{bmatrix}$$

$$N(2) = LN(1) = \begin{bmatrix} 0 & 0 & 4.6 & 3.7 \\ .7 & 0 & 0 & 0 \\ 0 & .5 & 0 & 0 \\ 0 & 0 & .1 & 0 \end{bmatrix} \begin{bmatrix} 1335 \\ 1050 \\ 250 \\ 25 \end{bmatrix} = \begin{bmatrix} 1242.5 \\ 934.5 \\ 525 \\ 25 \end{bmatrix} \approx \boxed{\begin{bmatrix} 1243 \\ 935 \\ 525 \\ 25 \end{bmatrix}}$$