MATH 17B

SOLUTIONS - SEC. 9.3

(56) $A = \begin{bmatrix} -3 & -1/2 \\ 7 & 3/2 \end{bmatrix}$

$\det(A - \lambda I) = \begin{vmatrix} -3-\lambda & -1/2 \\ 7 & \frac{3}{2}-\lambda \end{vmatrix} = (-3-\lambda)(\frac{3}{2}-\lambda) - (-\frac{7}{2}) = \lambda^2 + \frac{3}{2}\lambda - 1 = 0$ so

$2\lambda^2 + 3\lambda - 2 = 0$, $(2\lambda - 1)(\lambda + 2) = 0$, $\boxed{\lambda = \frac{1}{2}}$ OR $\boxed{\lambda = -2}$

a) $\lambda = \frac{1}{2}$: $(A - \frac{1}{2}I)x = 0$ GIVES $\begin{bmatrix} -\frac{7}{2} & -\frac{1}{2} & 0 \\ 7 & 1 & 0 \end{bmatrix} \longrightarrow \begin{array}{c} \begin{array}{ccc} x & y & \end{array} \\ \begin{bmatrix} 1 & 1/7 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$ LET $y = 7t$, SO $x = -t$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -t \\ 7t \end{bmatrix} = t\begin{bmatrix} -1 \\ 7 \end{bmatrix}$, SO $\boxed{\begin{bmatrix} -1 \\ 7 \end{bmatrix}}$ IS AN EIGENVECTOR FOR $\lambda = \frac{1}{2}$

b) $\lambda = -2$: $(A - (-2)I)x = 0$ GIVES $\begin{bmatrix} -1 & -1/2 & 0 \\ 7 & 7/2 & 0 \end{bmatrix} \longrightarrow \begin{array}{c} \begin{array}{ccc} x & y & \end{array} \\ \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$ LET $y = 2t$, SO $x = -t$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -t \\ 2t \end{bmatrix} = t\begin{bmatrix} -1 \\ 2 \end{bmatrix}$, SO $\boxed{\begin{bmatrix} -1 \\ 2 \end{bmatrix}}$ IS AN EIGENVECTOR FOR $\lambda = -2$

(63) $A = \begin{bmatrix} 2 & 4 \\ -2 & -3 \end{bmatrix}$ $\quad tr(A) = -1 < 0$ AND $\det(A) = 2 > 0$, SO THE REAL PARTS OF BOTH EIGENVALUES ARE NEGATIVE.

(65) $A = \begin{bmatrix} 4 & 4 \\ -4 & -3 \end{bmatrix}$ $\quad tr(A) = 1 > 0$, SO AT LEAST ONE EIGENVALUE HAS A REAL PART WHICH IS NOT NEGATIVE.

(66) $A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$ $\quad \det(A) = -2 < 0$, SO AT LEAST ONE EIGENVALUE HAS A REAL PART WHICH IS NOT NEGATIVE.

(67) $A = \begin{bmatrix} 2 & -5 \\ 2 & -3 \end{bmatrix}$ $\quad tr(A) = -1 < 0$ AND $\det(A) = 4 > 0$, SO THE REAL PARTS OF BOTH EIGENVALUES ARE NEGATIVE.

(69) $A = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$

a) $A\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, SO $u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ IS AN EIGENVECTOR FOR $\lambda_1 = -1$.

$A\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 3 \end{bmatrix}$, SO $u_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ IS AN EIGENVECTOR FOR $\lambda_2 = 2$.

u_1 AND u_2 ARE LINEARLY INDEPENDENT SINCE THEY ARE NOT MULTIPLES OF EACH OTHER.

b) $\begin{bmatrix} -1 \\ 3 \end{bmatrix} = c_1\begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ GIVES $\begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, SO

$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 3 & 3 \end{bmatrix} \xrightarrow{\frac{1}{3}R_2} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{-R_2+R_1} \begin{array}{c} \begin{array}{ccc} c_1 & c_2 & \end{array} \\ \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$ SO $c_1 = -2$ AND $c_2 = 1$.

c) $A^{20}x = A^{20}(-2u_1 + u_2) = -2(A^{20}u_1) + (A^{20}u_2) = -2(-1)^{20}u_1 + 2^{20}u_2 = -2u_1 + 2^{20}u_2$

$= -2(-1)^{20}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2^{20}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \boxed{\begin{bmatrix} 2^{20} - 2 \\ 3(2^{20}) \end{bmatrix}}$

MATH 17B SOLUTIONS – Sec. 9.3

(10) $A = \begin{bmatrix} -2 & 1 \\ -4 & 3 \end{bmatrix}$

a) $A\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 4 \end{bmatrix}$, so $u_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ is an eigenvector for $\lambda_1 = 2$.

$A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} = -1\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, so $u_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector for $\lambda_2 = -1$.

u_1 and u_2 are linearly independent, since they are not multiples of each other.

b) $c_1\begin{bmatrix} 1 \\ 4 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ gives $\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, so

$$\begin{bmatrix} 1 & 1 & -1 \\ 4 & 1 & 2 \end{bmatrix} \xrightarrow{-4R_1+R_2} \begin{bmatrix} 1 & 1 & -1 \\ 0 & -3 & 6 \end{bmatrix} \xrightarrow{-\frac{1}{3}R_2} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix} \xrightarrow{-R_2+R_1} \begin{array}{c} \begin{matrix} c_1 & c_2 & \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \end{bmatrix} \end{array}$$

so $\underline{c_1 = 1}$ and $\underline{c_2 = -2}$

c) $\underline{A^{10}x} = A^{10}(u_1 - 2u_2) = (A^{10}u_1) - 2(A^{10}u_2) = \underline{2^{10}u_1 - 2(-1)^{10}u_2} = \underline{2^{10}u_1 - 2u_2}$

$$= 2^{10}\begin{bmatrix} 1 \\ 4 \end{bmatrix} - 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2^{10} - 2 \\ 4(2^{10}) - 2 \end{bmatrix} = \boxed{\begin{bmatrix} 1022 \\ 4094 \end{bmatrix}}$$

(12) $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$

$\det(A - \lambda I) = \begin{vmatrix} 4-\lambda & -3 \\ 2 & -1-\lambda \end{vmatrix} = (4-\lambda)(-1-\lambda) - (-6) = \lambda^2 - 3\lambda + 2 = (\lambda - 1)(\lambda - 2) = 0$ if $\boxed{\lambda = 1}$ or $\boxed{\lambda = 2}$

a) $\underline{\lambda = 1}$: $(A - I)x = 0$ gives $\begin{bmatrix} 3 & -3 & 0 \\ 2 & -2 & 0 \end{bmatrix} \longrightarrow \begin{array}{c} \begin{matrix} x & y & \end{matrix} \\ \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$ Let $\underline{y = t}$, so $x = t$:

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t \\ t \end{bmatrix} = t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, so $\underline{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$ is an eigenvector for $\lambda = 1$.

b) $\underline{\lambda = 2}$: $(A - 2I)x = 0$ gives $\begin{bmatrix} 2 & -3 & 0 \\ 2 & -3 & 0 \end{bmatrix} \longrightarrow \begin{array}{c} \begin{matrix} x & y & \end{matrix} \\ \begin{bmatrix} 1 & -3/2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$ Let $\underline{y = 2t}$, so $x = 3t$:

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3t \\ 2t \end{bmatrix} = t\begin{bmatrix} 3 \\ 2 \end{bmatrix}$, so $\underline{\begin{bmatrix} 3 \\ 2 \end{bmatrix}}$ is an eigenvector for $\lambda = 2$,

$c_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$ gives $\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$, so $\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} -4 \\ -2 \end{bmatrix}$

$$= \frac{1}{-1}\begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -4 \\ -2 \end{bmatrix} = \underline{\begin{bmatrix} 2 \\ -2 \end{bmatrix}}$$

Then $A^{30}\begin{bmatrix} -4 \\ -2 \end{bmatrix} = A^{30}(2u_1 - 2u_2) = 2(A^{30}u_1) - 2(A^{30}u_2)$

$$= \underline{+2(1)^{30}u_1 - 2(2^{30}u_2)} = \underline{2u_1 - 2^{31}u_2}$$

$$= 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 2^{31}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \boxed{\begin{bmatrix} -3(2^{31}) + 2 \\ -2(2^{31}) + 2 \end{bmatrix}} = \boxed{\begin{bmatrix} 2 - 3(2^{31}) \\ 2 - 2^{32} \end{bmatrix}}$$

SOLUTIONS – SEC. 9.3 AND CH. 9 RE

9.3 – (13) $A = \begin{bmatrix} 5 & 7 \\ -2 & -4 \end{bmatrix}$

$$\det(A - \lambda I) = \begin{vmatrix} 5-\lambda & 7 \\ -2 & -4-\lambda \end{vmatrix} = (5-\lambda)(-4-\lambda) - (-14) = \lambda^2 - \lambda - 6 = (\lambda - 3)(\lambda + 2) = 0$$

IF $\underline{\lambda = 3}$ OR $\underline{\lambda = -2}$.

a) $\underline{\lambda = 3}$: $(A - 3I)x = 0$ GIVES

$$\begin{bmatrix} 2 & 7 & 0 \\ -2 & -7 & 0 \end{bmatrix} \longrightarrow \begin{array}{c} \begin{array}{ccc} x & y & \end{array} \\ \begin{bmatrix} 1 & 7/2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

LET $\underline{Y = 2T}$, SO $X = -7T$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7t \\ 2t \end{bmatrix} = t\begin{bmatrix} -7 \\ 2 \end{bmatrix}, \text{ SO } \underline{\begin{bmatrix} -7 \\ 2 \end{bmatrix}} \text{ IS AN EIGENVECTOR FOR } \lambda = 3.$$

b) $\underline{\lambda = -2}$: $(A - (-2)I)x = 0$ GIVES

$$\begin{bmatrix} 7 & 7 & 0 \\ -2 & -2 & 0 \end{bmatrix} \longrightarrow \begin{array}{c} \begin{array}{ccc} x & y & \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

LET $\underline{Y = T}$, SO $X = -T$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -t \\ t \end{bmatrix} = t\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \text{ SO } \underline{\begin{bmatrix} -1 \\ 1 \end{bmatrix}} \text{ IS AN EIGENVECTOR FOR } \lambda = -2.$$

$$c_1\begin{bmatrix} -7 \\ 2 \end{bmatrix} + c_2\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix} \text{ GIVES } \begin{bmatrix} -7 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}, \text{ SO}$$

$$\begin{bmatrix} -7 & -1 & -3 \\ 2 & 1 & -2 \end{bmatrix} \xrightarrow[-R_1]{3R_2 + R_1} \begin{bmatrix} 1 & -2 & 9 \\ 2 & 1 & -2 \end{bmatrix} \xrightarrow{-2R_1 + R_2} \begin{bmatrix} 1 & -2 & 9 \\ 0 & 5 & -20 \end{bmatrix} \xrightarrow[2R_2 + R_1]{\frac{1}{5}R_2} \begin{array}{c} \begin{array}{ccc} c_1 & c_2 & \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -4 \end{bmatrix} \end{array}$$

SO $c_1 = 1$, $\underline{c_2 = -4}$

$$A^{20}\begin{bmatrix} -3 \\ -2 \end{bmatrix} = A^{20}(u_1 - 4u_2) = \underline{3^{20}u_1 - 4(-2)^{20}u_2} = \underline{3^{20}u_1 - 4(2^{20})u_2}$$

$$= 3^{20}\begin{bmatrix} -7 \\ 2 \end{bmatrix} - 2^{22}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 2^{22} - 7(3^{20}) \\ 2(3^{20}) - 2^{22} \end{bmatrix}}$$

CH. 9 RE – (5) $AB = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$ AND $A^{-1} = \begin{bmatrix} 4 & -1 \\ 8 & -1 \end{bmatrix}$

$$B = A^{-1}(AB) = \begin{bmatrix} 4 & -1 \\ 8 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} = \boxed{\begin{bmatrix} -2 & 5 \\ -2 & 9 \end{bmatrix}}$$

(6) $(AB)^{-1} = \begin{bmatrix} -1 & 3 \\ 0 & 2 \end{bmatrix}$ AND $B = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$

$$A^{-1} = B(B^{-1}A^{-1}) = B(AB)^{-1} = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ -1 & 9 \end{bmatrix},$$

$$\text{SO } A = (A^{-1})^{-1} = \frac{1}{-4}\begin{bmatrix} 9 & 4 \\ 1 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} -9/4 & -1 \\ -1/4 & 0 \end{bmatrix}}$$

OR $AB = ((AB)^{-1})^{-1} = \frac{1}{-2}\begin{bmatrix} 2 & -3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 3/2 \\ 0 & +1/2 \end{bmatrix}$ AND $B^{-1} = \frac{1}{2}\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 3/2 & 1 \\ -1/2 & 0 \end{bmatrix}$,

$$\text{SO } A = (AB)B^{-1} = \begin{bmatrix} -1 & 3/2 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} 3/2 & 1 \\ -1/2 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} -9/4 & -1 \\ -1/4 & 0 \end{bmatrix}}$$
