MATH 17B SOLUTIONS – SEC. 8.1

① $\frac{dy}{dx} = x + \sin x$, $y_0 = 0$ for $x_0 = 0$

$y = \int (x + \sin x)\,dx = \underline{\frac{x^2}{2} - \cos x + C}$ IF $x = 0$, $0 - 1 + C = 0$ SO $\underline{C = 1}$ AND

$\boxed{y = \frac{x^2}{2} - \cos x + 1}$

⑤ $\frac{dx}{dt} = \frac{1}{1-t}$, $x(0) = 2$

$x = \int \frac{1}{1-t}\,dt = (-)\int \frac{(-1)}{1-t}\,dt = \underline{-\ln|1-t| + C}$ IF $t = 0$, $-\ln \overset{0}{\downarrow} 1 + C = 2$ SO $\underline{C = 2}$

AND $\boxed{x = -\ln|1-t| + 2}$ OR $\boxed{x = -\ln(1-t) + 2}$ FOR $t < 1$

⑦ $\frac{ds}{dt} = \sqrt{3t+1}$, $s(0) = 1$

$s = \int \sqrt{3t+1}\,dt$ LET $\underline{u = 3t+1}$, $du = 3\,dt$

$= \frac{1}{3}\int \sqrt{3t+1}\cdot 3\,dt = \frac{1}{3}\int \sqrt{u}\,du = \frac{1}{3}\int u^{1/2}\,du = \frac{1}{3}\left[\frac{2}{3}u^{3/2}\right] + C = \underline{\frac{2}{9}(3t+1)^{3/2} + C}$

WHEN $t = 0$, $\frac{2}{9}\cdot 1^{3/2} + C = \frac{2}{9} + C = 1$ SO $\underline{C = \frac{7}{9}}$ AND

$\boxed{s = \frac{2}{9}(3t+1)^{3/2} + \frac{7}{9}}$

⑧ $\frac{dh}{dt} = 5 - 16t^2$, $h(3) = -11$

$h = \int (5 - 16t^2)\,dt = \underline{5t - \frac{16}{3}t^3 + C}$

WHEN $t = 3$, $5(3) - \frac{16}{3}(3)^3 + C = 15 - 144 + C = -11$ SO $\underline{C = 118}$ AND

$\boxed{h = 5t - \frac{16}{3}t^3 + 118}$

⑩ $\frac{dP}{dt} = 3t + 1$, $P(0) = 0$

$P = \int (3t+1)\,dt = \underline{\frac{3}{2}t^2 + t + C}$

WHEN $t = 0$, $0 + C = 0$ SO $\underline{C = 0}$ AND

$\boxed{P = \frac{3}{2}t^2 + t}$

(11) $\frac{dY}{dX} = 3Y$, $Y_0 = 2$ WHEN $X_0 = 0$

$\int \frac{1}{Y}\,dY = \int 3\,dX$, $\quad \text{LN } Y = 3X + C$, $\quad Y = e^{3X+C} = e^C \cdot e^{3X} = Ae^{3X}$

WHEN $X = 0$, $Y = 2$; SO $2 = A \cdot 1$ AND $A = 2$! $\quad \boxed{Y = 2e^{3X}}$

REMARK SINCE THIS IS THE DE FOR EXPONENTIAL GROWTH, WE COULD ALSO GET THIS ANSWER FROM THE EXPONENTIAL GROWTH FORMULA.

(15) $\frac{dh}{ds} = 2h + 1$, $h(0) = 4$

$\int \frac{1}{2h+1}\,dh = \int ds$, $\quad \frac{1}{2}\int \frac{2}{2h+1}\,dh = s + C$, $\quad \frac{1}{2}\text{LN}(2h+1) = s + C$, $\quad \text{LN}(2h+1) = 2s + D$

$2h + 1 = e^{2s+D} = e^D e^{2s} = Ae^{2s}$, $\quad 2h = Ae^{2s} - 1$, $\quad h = \frac{1}{2}(Ae^{2s} - 1)$

WHEN $s = 0$, $h = 4$: $\quad 4 = \frac{1}{2}(A \cdot 1 - 1)$ SO $A = 9$! $\quad \boxed{h = \frac{1}{2}(9e^{2s} - 1)}$

(16) $\frac{dN}{dT} = 5 - N$, $N(2) = 3$

$\int \frac{1}{5-N}\,dN = \int dT$, $\quad -\int \frac{-1}{5-N}\,dN = T + C$, $\quad -\text{LN}(5-N) = T + C$, $\quad \text{LN}(5-N) = -T - C$,

$5 - N = e^{-T-C} = e^{-C}e^{-T} = Ae^{-T}$, $\quad N = 5 - Ae^{-T}$

WHEN $T = 2$, $N = 3$; SO $3 = 5 - Ae^{-2}$, $\quad Ae^{-2} = 2$, $\quad A = 2e^2$

SO $\boxed{N = 5 - (2e^2)e^{-T} = 5 - 2e^{2-T}}$

(17) $\frac{dN}{dT} = .3N(T)$, $N(0) = 20$

$\int \frac{1}{N}\,dN = \int .3\,dT$, $\quad \text{LN } N = .3T + C$, $\quad N = e^{.3T+C} = e^C e^{.3T} = Ae^{.3T}$

WHEN $T = 0$, $N = 20$; SO $20 = A \cdot 1 = A$ AND $\boxed{N = 20e^{.3T}}$

REMARK WE COULD ALSO GET THIS ANSWER FROM THE EXPONENTIAL GROWTH FORMULA.

$N(5) = \boxed{20e^{1.5}} \approx 90$

(19) a) $\frac{1}{N} \cdot \frac{dN}{dT} = r$ SO $\int \frac{1}{N}\,dN = \int r\,dT$, $\quad \text{LN } N = rT + C$, $\quad N = e^{rT+C} = e^C e^{rT} = Ae^{rT}$

WHERE $T = 0$ GIVES $N(0) = A \cdot 1 = A$, SO $\boxed{N = N_0 e^{rT}}$

b) TAKING LOGARITHMS GIVES $\text{LOG } N = \text{LOG } N_0 + (rT)\,\text{LOG } e$ OR $\text{LOG } N = \text{LOG } N_0 + (r\,\text{LOG } e)T$

IF WE GRAPH N AS A FUNCTION OF T ON SEMILOG PAPER,

$r = \frac{m}{\text{LOG } e}$ WHERE m IS THE SLOPE OF THE LINE.

c) PLOT THE DATA ON SEMILOG PAPER, FIND THE SLOPE m OF THE RESULTING LINE,

AND THEN USE $r = \frac{m}{\text{LOG } e}$ (WHERE $\text{LOG } e \approx .434$)

(20) a) $\frac{dW}{dT} = -\lambda W$, $W(0) = W_0$ SO $\boxed{W = W_0 e^{-\lambda T}}$ BY THE EXPONENTIAL GROWTH/DECAY FORMULA.

b) $W = 123e^{-\lambda T}$ SINCE $W_0 = 123$, AND $W(5) = 20$ GIVES $123e^{-5\lambda} = 20$, $\quad e^{-5\lambda} = \frac{20}{123}$,

$-5\lambda = \text{LN }\frac{20}{123}$, $\quad \lambda = \boxed{-\frac{1}{5}\text{LN }\frac{20}{123}} = \boxed{\frac{1}{5}\text{LN }\frac{123}{20}}$ $\quad$ (SINCE $\text{LN }\frac{1}{T} = -\text{LN } T$)

WHEN $W = \frac{1}{2}(123)$, $\quad \cancel{123}\,e^{-\lambda T} = \frac{1}{2}(\cancel{123})$, $\quad e^{-\lambda T} = \frac{1}{2}$, $\quad -\lambda T = \text{LN }\frac{1}{2}$, $\quad T = \frac{\text{LN }\frac{1}{2}}{-\lambda}$

SO $T = \boxed{(5\,\text{LN }\tfrac{1}{2})/(\text{LN }\tfrac{20}{123})\text{ MIN}} = \boxed{5\left(\frac{\text{LN } 2}{\text{LN }\frac{123}{20}}\right)\text{ MIN}} \approx 1.9\text{ MIN}$