MATH 17B — SOLUTIONS – SEC. 5.8, 6.2, 6.3

5.8 – (72) $\frac{dN}{dT} = T^{-1/3}$, $T > 0$ AND $N(0) = 60$

$N = \int T^{-1/3}\,dT = \frac{T^{2/3}}{2/3} + C = \frac{3}{2}T^{2/3} + C$ AND $N(0) = 60 \Rightarrow C = 60$,

so $\boxed{N(T) = \frac{3}{2}T^{2/3} + 60}$

(73) $\frac{dL}{dx} = e^{-.1x}$, $x \geq 0$ AND $L_\infty = \lim_{x\to\infty} L(x) = 25$

$L = \int e^{-.1x}\,dx = \int e^{-\frac{1}{10}x}\,dx = \frac{1}{-\frac{1}{10}} e^{-\frac{1}{10}x} + C = -10e^{-\frac{1}{10}x} + C$, so

$L_\infty = \lim_{x\to\infty}\left(-10e^{-\frac{1}{10}x} + C\right) = \lim_{x\to\infty}\left(\frac{-10}{e^{.1x}} + C\right) = 0 + C = 25 \Rightarrow C = 25$.

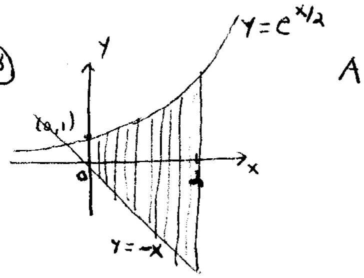
THEN $L(x) = -10e^{-.1x} + 25$, so $L(0) = -10(1) + 25 = \boxed{15}$

6.2 – (105) $\int_0^{\pi/4} \sin 2x\,dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/4} = -\frac{1}{2}\left(\cos\frac{\pi}{2} - \cos 0\right) = -\frac{1}{2}(0-1) = \boxed{\frac{1}{2}}$

(121) $\int_{-2}^{-1} \frac{1}{1-u}\,du = -\int_{-2}^{-1} \frac{-1}{1-u}\,du = -\left[\ln|1-u|\right]_{-2}^{-1} = -(\ln 2 - \ln 3)$

$= \ln 3 - \ln 2 = \boxed{\ln\frac{3}{2}}$

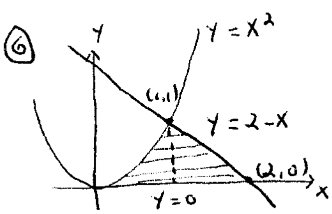
6.3 – (3)



$A = \int_0^2 \left(e^{x/2} - (-x)\right)dx = \left[2e^{x/2} + \frac{x^2}{2}\right]_0^2$

$= (2e + 2) - (2 + 0) = \boxed{2e}$

(6)



$A = \int_0^1 (2 - y - \sqrt{y})\,dy$ (SINCE $y = 2 - x$ GIVES $x = 2 - y$ AND $y = x^2$ GIVES $x = \pm\sqrt{y}$)

$= \left[2y - \frac{y^2}{2} - \frac{2}{3}y^{3/2}\right]_0^1 = \left(2 - \frac{1}{2} - \frac{2}{3}\right) - 0 = \frac{3}{2} - \frac{2}{3} = \boxed{\frac{5}{6}}$

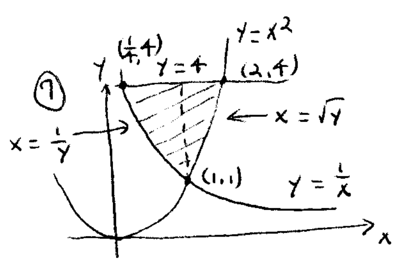
$x^2 = 2 - x$,
$x^2 + x - 2 = 0$, $(x+2)(x-1) = 0$,
$x = -2$, $x = 1$

OR $A = \int_0^1 x^2\,dx + \int_1^2 (2 - x)\,dx$

$= \left[\frac{1}{3}x^3\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2$

$= \frac{1}{3}(1 - 0) + (4 - 2) - \left(2 - \frac{1}{2}\right) = \frac{1}{3} + 2 - \frac{3}{2} = \boxed{\frac{5}{6}}$

(7)



$A = \int_1^4 \left(\sqrt{y} - \frac{1}{y}\right)dy = \left[\frac{2}{3}y^{3/2} - \ln y\right]_1^4$

$= \left(\frac{2}{3}\cdot 4^{3/2} - \ln 4\right) - \left(\frac{2}{3}\cdot 1 - \underset{0}{\cancel{\ln 1}}\right)$

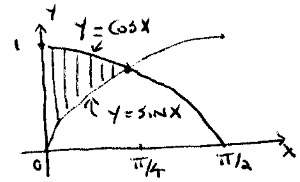
$= \frac{2}{3}\cdot 8 - \ln 4 - \frac{2}{3} = \boxed{\frac{14}{3} - \ln 4}$

$x^2 = \frac{1}{x}$, $x^3 = 1$, $x = 1$

OR $A = \int_{1/4}^1 \left(4 - \frac{1}{x}\right)dx + \int_1^2 (4 - x^2)\,dx = \left[4x - \ln x\right]_{1/4}^1 + \left[4x - \frac{x^3}{3}\right]_1^2$

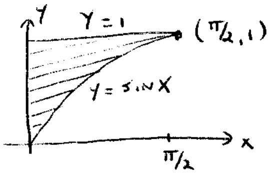
$= (4 - \underset{0}{\cancel{\ln 1}}) - \left(1 - \ln\frac{1}{4}\right) + \left(8 - \frac{8}{3}\right) - \left(4 - \frac{1}{3}\right) = \frac{14}{3} + \ln\frac{1}{4} = \boxed{\frac{14}{3} - \ln 4}$

(8)



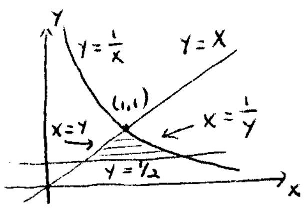
$$A = \int_0^{\pi/4} (\cos x - \sin x)\, dx = \Big[\sin x - (-\cos x)\Big]_0^{\pi/4}$$
$$= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - (0 + 1) = \boxed{\sqrt{2} - 1}$$

(9)



$$A = \int_0^{\pi/2} (1 - \sin x)\, dx = \Big[x + \cos x\Big]_0^{\pi/2} = \left(\frac{\pi}{2} + 0\right) - (0 + 1) = \boxed{\frac{\pi}{2} - 1}$$

(14)

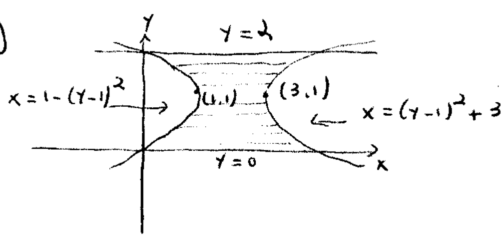


$$A = \int_{1/2}^{1} \left(\frac{1}{y} - y\right) dy = \left[\ln y - \frac{y^2}{2}\right]_{1/2}^{1} = (\underbrace{\cancel{\ln 1}}_{0} - \tfrac{1}{2}) - \left(\ln \tfrac{1}{2} - \tfrac{1}{8}\right)$$
$$= -\ln\frac{1}{2} - \frac{3}{8} = \boxed{\ln 2 - \frac{3}{8}}$$

[CHECK: $A = \int_{1/2}^{1} (x - \frac{1}{2})\, dx + \int_1^2 \left(\frac{1}{x} - \frac{1}{2}\right) dx = \left[\frac{x^2}{2} - \frac{1}{2}x\right]_{1/2}^{1} + \left[\ln x - \frac{1}{2}x\right]_1^2$

$= \cancel{\left(\frac{1}{2} - \frac{1}{2}\right)} - \left(\frac{1}{8} - \frac{1}{4}\right) + (\ln 2 - 1) - (\underbrace{\cancel{\ln 1}}_{0} - \frac{1}{2}) = \boxed{\ln 2 - \frac{3}{8}}$]

(15)



$$A = \int_0^2 \Big(\big((y-1)^2 + 3\big) - \big(1 - (y-1)^2\big)\Big)\, dy$$
$$= \int_0^2 \left(2 + 2(y-1)^2\right) dy = 2\int_0^2 \left(1 + (y-1)^2\right) dy$$
$$= 2\int_0^2 (y^2 - 2y + 2)\, dy = 2\left[\frac{y^3}{3} - y^2 + 2y\right]_0^2$$
$$= 2\left(\left(\frac{8}{3} - 4 + 4\right) - 0\right) = \boxed{\frac{16}{3}}$$

OR using symmetry,

$$A = 2\int_0^1 \Big(\big((y-1)^2 + 3\big) - \big(1 - (y-1)^2\big)\Big)\, dy = 2\int_0^1 \left(2 + 2(y-1)^2\right) dy$$
$$= 4\int_0^1 \left(1 + (y-1)^2\right) dy = 4\left[y + \frac{1}{3}(y-1)^3\right]_0^1 = 4\left((1 + 0) - \left(0 - \frac{1}{3}\right)\right) = 4 \cdot \frac{4}{3} = \boxed{\frac{16}{3}}$$

(17) $\frac{dN}{dt} = e^{-t}$, $N(0) = 100$

a) $N = \int e^{-t}\, dt = \underline{-e^{-t} + C}$, so $N(0) = -1 + C = 100 \Rightarrow \underline{C = 101}$ AND

$\boxed{N(t) = -e^{-t} + 101}$

b) $N(5) - N(0) = \int_0^5 \frac{dN}{dt}\, dt = \int_0^5 e^{-t}\, dt = \left[-e^{-t}\right]_0^5 = -e^{-5} - (-1) = \boxed{1 - e^{-5}}$

(OR $N(5) - N(0) = (-e^{-5} + 101) - (100) = \boxed{1 - e^{-5}}$)

c) $N(t) - N(0) = \int_0^t N'(u)\, du = \underline{\int_0^t e^{-u}\, du}$