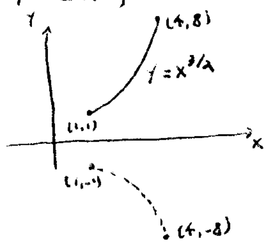
MATH 17B SOLUTIONS – SEC. 6.3

(55) $y^2 = x^3, \quad 1 \le x \le 4$



$$s = \int_1^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_1^4 \sqrt{1+\left(\tfrac{3}{2}x^{1/2}\right)^2}\,dx = \int_1^4 \sqrt{1+\tfrac{9}{4}x}\,dx = \int_1^4 \sqrt{\frac{4+9x}{4}}\,dx$$

$$= \frac{1}{2}\int_1^4 \sqrt{4+9x}\,dx \qquad \text{LET } u = 4+9x, \quad du = 9\,dx \qquad \text{IF } x=1,\ u=13; \quad x=4,\ u=40$$

$$= \frac{1}{2}\cdot\frac{1}{9}\int_1^4 \sqrt{4+9x}\cdot 9\,dx = \frac{1}{18}\int_{13}^{40}\sqrt{u}\,du = \frac{1}{18}\int_{13}^{40} u^{1/2}\,du = \frac{1}{18}\left[\frac{2}{3}u^{3/2}\right]_{13}^{40}$$

$$= \boxed{\frac{1}{27}\left(40^{3/2} - 13^{3/2}\right)} = \boxed{\frac{1}{27}\left(8\left(10^{3/2}\right) - 13^{3/2}\right)}$$

REMARK: NOTICE THAT WE HAVE ONLY FOUND THE LENGTH OF THE PORTION OF THE CURVE WHICH IS ABOVE THE X-AXIS.

(57) $y = \dfrac{x^3}{6} + \dfrac{1}{2x}, \quad 1 \le x \le 3 \qquad y' = \dfrac{x^2}{2} - \dfrac{1}{2}x^{-2} = \dfrac{x^2}{2} - \dfrac{1}{2x^2}$, so

$$s = \int_1^3 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_1^3 \sqrt{1+\left(\frac{x^2}{2}-\frac{1}{2x^2}\right)^2}\,dx = \int_1^3 \sqrt{1+\left(\frac{x^4}{4}-\frac{1}{2}+\frac{1}{4x^4}\right)}\,dx = \int_1^3 \sqrt{\frac{x^4}{4}+\frac{1}{2}+\frac{1}{4x^4}}\,dx$$

$$= \int_1^3 \sqrt{\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)^2}\,dx = \int_1^3 \left(\frac{x^2}{2}+\frac{1}{2x^2}\right)dx = \left[\frac{x^3}{6}-\frac{1}{2x}\right]_1^3 = \left(\frac{27}{6}-\frac{1}{6}\right)-\left(\frac{1}{6}-\frac{1}{2}\right) = \boxed{\frac{14}{3}}$$

$\left(\text{SINCE } \dfrac{x^2}{2}+\dfrac{1}{2x^2} > 0\right)$

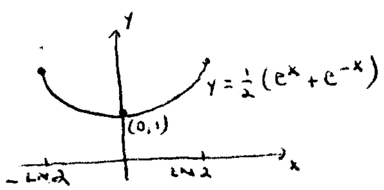
(58) $y = \dfrac{x^4}{4} + \dfrac{1}{8x^2}, \quad 2 \le x \le 4 \qquad y' = x^3 - \dfrac{1}{4}x^{-3} = x^3 - \dfrac{1}{4x^3}$, so

$$s = \int_2^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_2^4 \sqrt{1+\left(x^3-\frac{1}{4x^3}\right)^2}\,dx = \int_2^4 \sqrt{1+\left(x^6-\frac{1}{2}+\frac{1}{16x^6}\right)}\,dx = \int_2^4 \sqrt{x^6+\frac{1}{2}+\frac{1}{16x^6}}\,dx$$

$$= \int_2^4 \sqrt{\left(x^3+\frac{1}{4x^3}\right)^2}\,dx = \int_2^4 \left|x^3+\frac{1}{4x^3}\right|dx = \int_2^4 \left(x^3+\frac{1}{4x^3}\right)dx \longleftarrow \left(\text{SINCE } x^3+\frac{1}{4x^3} > 0 \text{ FOR } 2 \le x \le 4\right)$$

$$= \left[\frac{x^4}{4}-\frac{1}{8x^2}\right]_2^4 = \left(64-\frac{1}{128}\right)-\left(4-\frac{1}{32}\right) = 60 + \frac{3}{128} = \boxed{\frac{7683}{128}}$$

(64) $y' = \dfrac{1}{2}\left(e^x - e^{-x}\right)$



(USING SYMMETRY)

$$s = \int_{-\ln 2}^{\ln 2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = 2\int_0^{\ln 2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = 2\int_0^{\ln 2}\sqrt{1+\frac{1}{4}\left(e^x-e^{-x}\right)^2}\,dx$$

$$= 2\int_0^{\ln 2}\sqrt{1+\frac{1}{4}\left(e^{2x}-2+e^{-2x}\right)}\,dx = 2\int_0^{\ln 2}\sqrt{1+\frac{1}{4}e^{2x}-\frac{1}{2}+\frac{1}{4}e^{-2x}}\,dx = 2\int_0^{\ln 2}\sqrt{\frac{1}{4}e^{2x}+\frac{1}{2}+\frac{1}{4}e^{-2x}}\,dx$$

$$= 2\int_0^{\ln 2}\sqrt{\frac{1}{4}\left(e^x+e^{-x}\right)^2}\,dx = \cancel{2}\int_0^{\ln 2}\frac{1}{\cancel{2}}\left(e^x+e^{-x}\right)dx = \left[e^x - e^{-x}\right]_0^{\ln 2} =$$

$\left(\text{SINCE } e^x + e^{-x} > 0\right)$

$$= \left(e^{\ln 2} - e^{-\ln 2}\right) - (1-1)$$

$$= 2 - \left(e^{\ln 2}\right)^{-1} = 2 - \frac{1}{2} = \boxed{\frac{3}{2}}$$

(21) $\displaystyle\int \frac{x-1}{1+4x-2x^2}\,dx$ LET $u = 1+4x-2x^2$, $du = (4-4x)\,dx = -4(x-1)\,dx$

$= \left(-\frac{1}{4}\right)\displaystyle\int \frac{1}{1+4x-2x^2}(-4)(x-1)\,dx = -\frac{1}{4}\int \frac{1}{u}\,du = -\frac{1}{4}\ln|u| + C = \boxed{-\frac{1}{4}\ln|1+4x-2x^2| + C}$

(23) $\displaystyle\int \frac{2x}{1+2x^2}\,dx$ LET $u = 1+2x^2$, $du = 4x\,dx$

$= \frac{2}{4}\displaystyle\int \frac{1}{1+2x^2}\cdot 4x\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \boxed{\frac{1}{2}\ln|1+2x^2| + C} = \boxed{\frac{1}{2}\ln(1+2x^2) + C}$

(SINCE $1+2x^2 > 0$ FOR ALL x)

(25) $\displaystyle\int 3x e^{x^2}\,dx$ LET $u = x^2$, $du = 2x\,dx$

$= \frac{3}{2}\displaystyle\int e^{x^2}\cdot 2x\,dx = \frac{3}{2}\int e^u\,du = \frac{3}{2}e^u + C = \boxed{\frac{3}{2}e^{x^2} + C}$

(31) $\displaystyle\int \tan x \sec^2 x\,dx$ LET $u = \tan x$, $du = \sec^2 x\,dx$

$= \displaystyle\int u\,du = \frac{u^2}{2} + C = \boxed{\frac{1}{2}\tan^2 x + C}$

OR LET $u = \sec x$, $du = \sec x \tan x\,dx$ TO GET

$\displaystyle\int \tan x \sec^2 x\,dx = \int \sec x \cdot \sec x \tan x\,dx = \int u\,du = \frac{u^2}{2} + C = \boxed{\frac{1}{2}\sec^2 x + C}$

(32) $\displaystyle\int \sin^3 x \cos x\,dx$ LET $u = \sin x$, $du = \cos x\,dx$

$= \displaystyle\int u^3\,du = \frac{u^4}{4} + C = \boxed{\frac{1}{4}\sin^4 x + C}$

(33) $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$ LET $u = \ln x$, $du = \frac{1}{x}\,dx$

$= \displaystyle\int (\ln x)^2\cdot\frac{1}{x}\,dx = \int u^2\,du = \frac{u^3}{3} + C = \boxed{\frac{1}{3}(\ln x)^3 + C}$

(35) $\displaystyle\int x^3\sqrt{5+x^2}\,dx$ LET $u = 5+x^2$, $du = 2x\,dx$ AND $x^2 = u-5$

$= \frac{1}{2}\displaystyle\int x^2\sqrt{5+x^2}\cdot 2x\,dx = \frac{1}{2}\int (u-5)\sqrt{u}\,du = \frac{1}{2}\int (u-5)u^{1/2}\,du = \frac{1}{2}\int (u^{3/2} - 5u^{1/2})\,du$

$= \frac{1}{2}\left[\frac{2}{5}u^{5/2} - \frac{10}{3}u^{3/2}\right] + C = \boxed{\frac{1}{5}(5+x^2)^{5/2} - \frac{5}{3}(5+x^2)^{3/2} + C}$

(37) $\displaystyle\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx$ LET $u = \sin x$, $du = \cos x\,dx$

$= \displaystyle\int \frac{1}{\sin x}\cdot\cos x\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|\sin x| + C}$