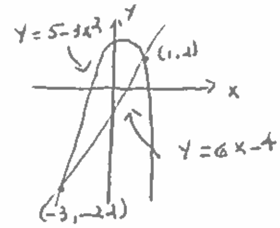
① $Y = \int_1^{x^2-3} \sqrt{u+8}\,du$, so $Y' = \sqrt{(x^2-3)+8}\cdot D_x(x^2-3) = \boxed{\sqrt{x^2+5}\cdot 2x}$

② $\int_0^3 x\sqrt{9-x^2}\,dx$ Let $u = 9-x^2$, $du = -2x\,dx$ if $x=0$, $u=9$; $x=3$, $u=0$

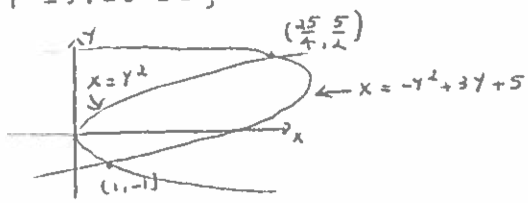
$= -\frac{1}{2}\int_0^3 \sqrt{9-x^2}\,(-2)x\,dx = -\frac{1}{2}\int_9^0 \sqrt{u}\,du = \frac{1}{2}\int_0^9 u^{1/2}\,du = \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_0^9 = \frac{1}{3}\left(9^{3/2}-0^{3/2}\right) = \frac{1}{3}\cdot 3^3 = \boxed{9}$

③ $A = \int_{-3}^1 \left((5-3x^2)-(6x-4)\right)dx = \int_{-3}^1 (9-3x^2-6x)\,dx = \left[9x - x^3 - 3x^2\right]_{-3}^1$

$= (9-1-3) - (-27-(-27)-27) = 5+27 = \boxed{32}$



④ $x = Y^2$ and $x = -Y^2+3Y+5$ intersect where $Y^2 = -Y^2+3Y+5$,
$2Y^2-3Y-5 = 0$, $(2Y-5)(Y+1) = 0$, $Y = 5/2$ or $Y = -1$



$A = \boxed{\int_{-1}^{5/2} \left((-Y^2+3Y+5)-Y^2\right)dY}$

$= \boxed{\int_{-1}^{5/2} (-2Y^2+3Y+5)\,dY}$

⑤ $\int_0^1 \frac{4x}{x^4+1}\,dx$ Let $u = x^2$, $du = 2x\,dx$ if $x=0$, $u=0$; $x=1$, $u=1$

$= 2\int_0^1 \frac{1}{(x^2)^2+1}\cdot 2x\,dx = 2\int_0^1 \frac{1}{u^2+1}\,du = 2\left[\tan^{-1}u\right]_0^1 = 2\left(\tan^{-1}1 - \tan^{-1}0\right) = 2\left(\frac{\pi}{4}-0\right) = \boxed{\frac{\pi}{2}}$

⑥ $\int_0^4 \frac{3x}{\sqrt{2x+1}}\,dx$ Let $u = \sqrt{2x+1}$, $x = \frac{1}{2}(u^2-1)$, $dx = u\,du$ if $x=0$, $u=1$; $x=4$, $u=3$

$= \int_1^3 \frac{3\cdot\frac{1}{2}(u^2-1)}{\cancel{u}}\cdot\cancel{u}\,du = \frac{3}{2}\int_1^3 (u^2-1)\,du = \frac{3}{2}\left[\frac{u^3}{3}-u\right]_1^3 = \frac{3}{2}\left((9-3)-\left(\frac{1}{3}-1\right)\right) = \frac{3}{2}\left(6+\frac{2}{3}\right) = \frac{3}{2}\cdot\frac{20}{3} = \boxed{10}$

OR Let $u = 2x+1$, $x = \frac{1}{2}(u-1)$, $dx = \frac{1}{2}\,du$ if $x=0$, $u=1$ and if $x=4$, $u=9$:

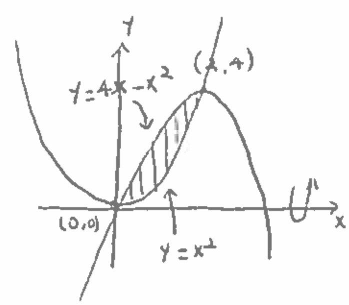
$\int_1^9 \frac{3\cdot\frac{1}{2}(u-1)}{\sqrt{u}}\cdot\frac{1}{2}\,du = \frac{3}{4}\int_1^9 \frac{u-1}{\sqrt{u}}\,du = \frac{3}{4}\int_1^9 \left(u^{1/2}-u^{-1/2}\right)du = \frac{3}{4}\left[\frac{2}{3}u^{3/2}-2u^{1/2}\right]_1^9$

$= \frac{3}{4}\left(\left(\frac{2}{3}\cdot 27 - 2\cdot 3\right)-\left(\frac{2}{3}-2\right)\right) = \frac{3}{4}\left(12-\left(-\frac{4}{3}\right)\right) = \frac{3}{4}\cdot\frac{40}{3} = \boxed{10}$

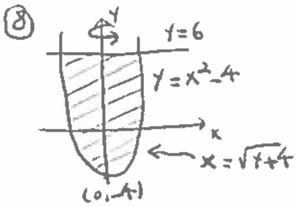
⑦ $V = \int_0^2 \pi\left((4x-x^2)^2-(x^2)^2\right)dx = \pi\int_0^2 \left(16x^2-8x^3+\cancel{x^4}-\cancel{x^4}\right)dx$

$= \pi\left[\frac{16}{3}x^3 - 2x^4\right]_0^2 = \pi\left(\frac{16}{3}\cdot 8 - 2\cdot 16\right) = 16\pi\left(\frac{8}{3}-2\right)$

$= 16\pi\cdot\frac{2}{3} = \boxed{\frac{32\pi}{3}}$



(8)



$$V=\int_{-4}^{6}\pi(f(y))^2\,dy=\int_{-4}^{6}\pi(\sqrt{y+4})^2\,dy=\pi\int_{-4}^{6}(y+4)\,dy=\pi\left[\frac{y^2}{2}+4y\right]_{-4}^{6}$$

$$=\pi\big((18+24)-(8-16)\big)=\pi(42+8)=\boxed{50\pi}$$

$\boxed{\text{OR}}$ use $u=y+4$, $du=dy$ if $y=-4$, $u=0$ if $y=6$, $u=10$ to get

$$V=\pi\int_{-4}^{6}(y+4)\,dy=\pi\int_{0}^{10}u\,du=\pi\left[\frac{u^2}{2}\right]_0^{10}=\frac{\pi}{2}(100)=\boxed{50\pi}$$

(9)

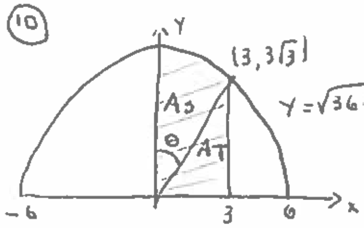
$$L=\int 20e^{-t/2}\,dt=20\left[-2e^{-t/2}\right]+C=\underline{-40e^{-t/2}+C}\qquad L(0)=-40\cdot 1+C=5\quad\text{so}\quad \underline{C=45}$$

and $\boxed{L(t)=-40e^{-t/2}+45}$

$$\underline{\lim_{t\to\infty}L(t)}=\lim_{t\to\infty}\left(-40e^{-t/2}+45\right)=\lim_{t\to\infty}\left(\frac{-40}{e^{t/2}}+45\right)$$

$$=0+45=\boxed{45\text{ cm}}$$

(10)



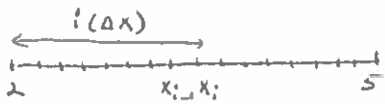
$$\int_0^3\sqrt{36-x^2}\,dx=A_S+A_T=\frac{1}{2}r^2\theta+\frac{1}{2}bh$$

$$=\frac{1}{2}\cdot 6^2\cdot\frac{\pi}{6}+\frac{1}{2}\cdot 3\cdot 3\sqrt{3}=\boxed{3\pi+\frac{9}{2}\sqrt{3}}$$

(where $\underline{\theta}=\tan^{-1}\frac{3}{3\sqrt{3}}=\tan^{-1}\frac{1}{\sqrt{3}}=\underline{\frac{\pi}{6}}$, or use $\underline{\theta}=\frac{\pi}{2}-\tan^{-1}\sqrt{3}=\frac{\pi}{2}-\frac{\pi}{3}=\underline{\frac{\pi}{6}}$)

(11)

$$\int_2^5(x^2+6x)\,dx=\lim_{\|P\|\to 0}\sum_{i=1}^{n}f(c_i)\,\Delta x_i$$



$$\underline{\Delta x}=\frac{5-2}{n}=\underline{\frac{3}{n}}\qquad \underline{c_i}=x_i=2+i(\Delta x)=\underline{2+\frac{3i}{n}}$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(2+\frac{3i}{n}\right)^2+6\left(2+\frac{3i}{n}\right)\right]\cdot\frac{3}{n}$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\left[16+\frac{30i}{n}+\frac{9i^2}{n^2}\right]\cdot\frac{3}{n}=\lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{48}{n}+\frac{90}{n^2}i+\frac{27}{n^3}i^2\right]$$

$$=\lim_{n\to\infty}\left[\sum_{i=1}^{n}\frac{48}{n}+\frac{90}{n^2}\sum_{i=1}^{n}i+\frac{27}{n^3}\sum_{i=1}^{n}i^2\right]=\lim_{n\to\infty}\left[48+\frac{90}{n^2}\cdot\frac{n(n+1)}{2}+\frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$

$$=\lim_{n\to\infty}\left[48+45\cdot\frac{n+1}{n}+\frac{9}{2}\cdot\frac{2n^2+3n+1}{n^2}\right]=48+45\cdot 1+\frac{9}{2}\cdot 2=\boxed{102}$$

(12)

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n+i}=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{1+\left(\frac{i}{n}\right)}\cdot\frac{1}{n}$$

$\leftarrow$ Riemann sum for $f(x)=\frac{1}{1+x}$ on $[0,1]$ using equal subintervals and right endpoints as sampling numbers so $\underline{c_i}=x_i=0+i(\Delta x)=\underline{\frac{i}{n}}$

$$=\int_0^1\frac{1}{1+x}\,dx=\Big[\ln(1+x)\Big]_0^1=\ln 2-\ln 1=\boxed{\ln 2}$$

$\boxed{\text{OR}}$ use

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n+i}=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{1+\left(\frac{i}{n}\right)}\cdot\frac{1}{n}$$

$\leftarrow$ Riemann sum for $f(x)=\frac{1}{x}$ on $[1,2]$ using equal subintervals and right endpoints as sampling numbers so $\underline{c_i}=x_i=1+i(\Delta x)=\underline{1+\frac{i}{n}}$

$$=\int_1^2\frac{1}{x}\,dx=\Big[\ln x\Big]_1^2=\ln 2-\ln 1=\boxed{\ln 2}$$