MATH 17B ANSWERS – TEST 1

① $$\int \frac{(x+1)(x+3)}{x}\,dx = \int \frac{x^2+4x+3}{x}\,dx = \int \left(x+4+\frac{3}{x}\right)dx = \boxed{\frac{x^2}{2}+4x+3\ln|x|+C}$$

② A) $$\int_0^4 \frac{10x}{\sqrt{x^2+9}}\,dx \qquad \text{Let } u = x^2+9,\ du = 2x\,dx \qquad \text{if } x=0,\ u=9;\ x=4,\ u=25$$

$$= \frac{10}{2}\int_0^4 \frac{1}{\sqrt{x^2+9}}\cdot 2x\,dx = 5\int_9^{25}\frac{1}{\sqrt{u}}\,du = 5\int_9^{25} u^{-1/2}\,du = 5\left[2u^{1/2}\right]_9^{25} = 10(5-3) = \boxed{20}$$

B) $$\int_2^6 \frac{6x}{\sqrt{2x-3}}\,dx \qquad \text{Let } u = \sqrt{2x-3},\quad x = \tfrac{1}{2}(u^2+3),\quad dx = \tfrac{1}{2}\cdot 2u\,du = u\,du \qquad \text{if } x=2,\ u=1;\ x=6,\ u=3$$

$$= \int_1^3 \frac{6\cdot\frac{1}{2}(u^2+3)}{\cancel{u}}\,\cancel{u}\,du = 3\int_1^3 (u^2+3)\,du = 3\left[\frac{u^3}{3}+3u\right]_1^3 = \left[u^3+9u\right]_1^3 = (27+27)-(1+9) = 54-10 = \boxed{44}$$

OR Let $u = 2x-3$, $x = \frac{1}{2}(u+3)$, $dx = \frac{1}{2}\,du$ if $x=2,\ u=1$; $x=6,\ u=9$!

$$\int_2^6 \frac{6x}{\sqrt{2x-3}}\,dx = \int_1^9 \frac{6\cdot\frac{1}{2}(u+3)}{\sqrt{u}}\cdot\frac{1}{2}\,du = \frac{3}{2}\int_1^9 \frac{u+3}{\sqrt{u}}\,du = \frac{3}{2}\int_1^9 (u+3)u^{-1/2}\,du$$

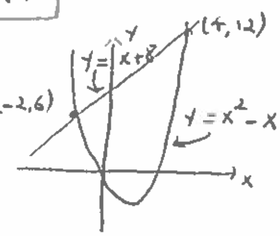
$$= \frac{3}{2}\int_1^9 \left(u^{1/2}+3u^{-1/2}\right)du = \frac{3}{2}\left[\frac{2}{3}u^{3/2}+6u^{1/2}\right]_1^9 = \left[u^{3/2}+9u^{1/2}\right]_1^9 = (27+27)-(1+9) = 54-10 = \boxed{44}$$

③ $$\frac{dL}{dT} = 4e^{-T/5}, \quad \text{so } L = \int 4e^{-T/5}\,dT = 4\left(-5e^{-T/5}\right)+C = \underline{-20e^{-T/5}+C}$$

$$L(0) = -20\cdot 1 + C = 4, \quad \text{so } \underline{C=24} \quad \text{and} \quad \boxed{L(T) = -20e^{-T/5}+24}$$

④ $$A = \int_{-2}^{4}\left(x+8-(x^2-x)\right)dx = \int_{-2}^{4}(2x+8-x^2)\,dx = \left[x^2+8x-\frac{x^3}{3}\right]_{-2}^{4}$$

$$= \left(16+32-\frac{64}{3}\right)-\left(4-16-\left(-\frac{8}{3}\right)\right) = 48-\frac{64}{3}+12-\frac{8}{3} = 60-24 = \boxed{36}$$



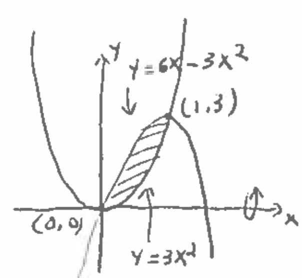
⑤ $$f_{av} = \frac{1}{6-0}\int_0^6 \frac{48T}{T^2+9}\,dT \qquad u = T^2+9,\ du = 2T\,dT$$

$$= \frac{48}{6}\cdot\frac{1}{2}\int_0^6 \frac{2T}{T^2+9}\,dT = 4\left[\ln(T^2+9)\right]_0^6 = 4\left(\ln 45 - \ln 9\right) = \boxed{4\ln 5 \text{ cm/sec}}$$

$\ln\frac{45}{9}$

⑥ $$V = \int_0^1 \pi\left((6x-3x^2)^2-(3x^2)^2\right)dx = \pi\int_0^1\left(36x^2-36x^3+\cancel{9x^4}-\cancel{9x^4}\right)dx$$

$$= \pi\left[12x^3-9x^4\right]_0^1 = \pi(12-9) = \boxed{3\pi}$$



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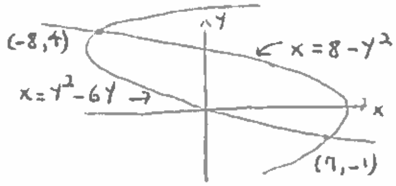
(7) $y^2-6y=8-y^2$, $2y^2-6y-8=0$, $y^2-3y-4=0$, $(y-4)(y+1)=0$, $\underline{y=4}$ or $\underline{y=-1}$



$$A=\boxed{\int_{-1}^{4}\left(8-y^2-(y^2-6y)\right)dy}=\boxed{\int_{-1}^{4}\left(8-2y^2+6y\right)dy}$$

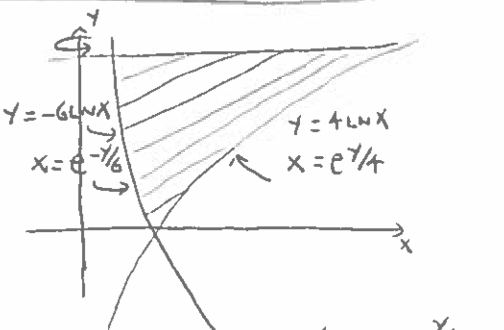
(8) $\int x^3\sqrt{x^2+4}\,dx$ $\qquad \dfrac{u=x^2+4}{du=2x\,dx} \longleftarrow x^2=u-4$

$$=\frac{1}{2}\int x^2\sqrt{x^2+4}\cdot 2x\,dx=\frac{1}{2}\int(u-4)\sqrt{u}\,du=\frac{1}{2}\int(u-4)u^{1/2}\,du=\frac{1}{2}\int\left(u^{3/2}-4u^{1/2}\right)du$$

$$=\frac{1}{2}\left[\frac{2}{5}u^{5/2}-\frac{8}{3}u^{3/2}\right]+C=\boxed{\frac{1}{5}(x^2+4)^{5/2}-\frac{4}{3}(x^2+4)^{3/2}+C}$$

(9) $$V=\int_0^6\pi\left((e^{y/4})^2-(e^{-y/6})^2\right)dy=\pi\int_0^6\left(e^{y/2}-e^{-y/3}\right)dy$$

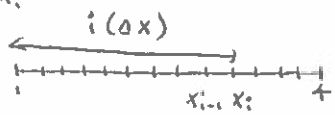
$$=\pi\left[2e^{y/2}+3e^{-y/3}\right]_0^6=\boxed{\pi\left(2e^3+3e^{-2}-5\right)}$$



$y=4\ln x$, $\ln x=\frac{y}{4}$, $\underline{x=e^{y/4}}$

$y=-6\ln x$, $\ln x=-\frac{y}{6}$, $\underline{x=e^{-y/6}}$

(10) $$\int_1^4(x^2+4x)\,dx=\lim_{\|P\|\to 0}\sum_{i=1}^{n}f(c_i)\,\Delta x_i$$



$\underline{\Delta x}=\frac{4-1}{n}=\frac{3}{n}$

$\underline{c_i}=x_i=1+i(\Delta x)=\underline{1+\frac{3i}{n}}$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(1+\frac{3i}{n}\right)^2+4\left(1+\frac{3i}{n}\right)\right]\cdot\frac{3}{n}$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\left[5+\frac{18i}{n}+\frac{9i^2}{n^2}\right]\cdot\frac{3}{n}$$

$$=\lim_{n\to\infty}\left[\sum_{i=1}^{n}5+\frac{18}{n}\sum_{i=1}^{n}i+\frac{9}{n^2}\sum_{i=1}^{n}i^2\right]\cdot\frac{3}{n}=\lim_{n\to\infty}\left[5n+\frac{18}{n}\cdot\frac{n(n+1)}{2}+\frac{9}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}\right]\cdot\frac{3}{n}$$

$$=\lim_{n\to\infty}\left[15+27\cdot\frac{n+1}{n}+\frac{9}{2}\cdot\frac{2n^2+3n+1}{n^2}\right]=15+27\cdot 1+\frac{9}{2}\cdot 2=15+27+9=\boxed{51}$$

(11) $$s=\int_{27/64}^{64/27}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\int_{27/64}^{64/27}\sqrt{1+(x^{-1/3})^2}\,dx=\int_{27/64}^{64/27}\sqrt{1+x^{-2/3}}\,dx=\int_{27/64}^{64/27}\sqrt{1+\frac{1}{x^{2/3}}}\,dx$$

$$=\int_{27/64}^{64/27}\sqrt{\frac{x^{2/3}+1}{x^{2/3}}}\,dx=\int_{27/64}^{64/27}\frac{\sqrt{x^{2/3}+1}}{x^{1/3}}\,dx$$

Let $\dfrac{u=x^{2/3}+1}{du=\frac{2}{3}x^{-1/3}\,dx}$

If $x=\frac{27}{64}$, $u=\left(\frac{3}{4}\right)^2+1=\frac{25}{16}$

$x=\frac{64}{27}$, $u=\left(\frac{4}{3}\right)^2+1=\frac{25}{9}$

$$=\frac{3}{2}\int_{27/64}^{64/27}\sqrt{x^{2/3}+1}\cdot\frac{2}{3}x^{-1/3}\,dx=\frac{3}{2}\int_{25/16}^{25/9}\sqrt{u}\,du=\frac{3}{2}\int_{25/16}^{25/9}u^{1/2}\,du$$

$$=\frac{3}{2}\left[\frac{2}{3}u^{3/2}\right]_{25/16}^{25/9}=\left(\frac{25}{9}\right)^{3/2}-\left(\frac{25}{16}\right)^{3/2}=\left(\frac{5}{3}\right)^3-\left(\frac{5}{4}\right)^3=\frac{125}{27}-\frac{125}{64}=125\left(\frac{64-27}{(27)(64)}\right)=\boxed{\frac{4625}{1728}}$$