(1) $\int x^2\cos 5x\,dx = \boxed{\frac{1}{5}x^2\sin 5x + \frac{2}{25}x\cos 5x - \frac{2}{125}\sin 5x + C}$ ←

u		dv
x^2	$+$	$\cos 5x\,dx$
$2x$	$-$	$\frac{1}{5}\sin 5x$
2	$+$	$-\frac{1}{25}\cos 5x$
0		$-\frac{1}{125}\sin 5x$

OR Let $u = x^2$, $dv = \cos 5x\,dx$

$du = 2x\,dx$, $v = \frac{1}{5}\sin 5x$

$= \frac{1}{5}x^2\sin 5x - \frac{2}{5}\int x\sin 5x\,dx$ Let $u = x$, $dv = \sin 5x\,dx$

$du = dx$, $v = -\frac{1}{5}\cos 5x$

$= \frac{1}{5}x^2\sin 5x - \frac{2}{5}\left[-\frac{1}{5}x\cos 5x - \left(-\frac{1}{5}\right)\int\cos 5x\,dx\right]$

$= \boxed{\frac{1}{5}x^2\sin 5x + \frac{2}{25}x\cos 5x - \frac{2}{25}\left(\frac{1}{5}\sin 5x\right) + C}$

$= \boxed{\frac{1}{5}x^2\sin 5x + \frac{2}{25}x\cos 5x - \frac{2}{125}\sin 5x + C}$

(2) $\int_1^9 \frac{1}{x^3+2}\,dx \approx \boxed{\frac{2}{3}\left[\frac{1}{1^3+2} + 4\cdot\frac{1}{3^3+2} + 2\cdot\frac{1}{5^3+2} + 4\cdot\frac{1}{7^3+2} + \frac{1}{9^3+2}\right]}$

(SIMPSON'S RULE WITH $n = 4$: 1 3 5 7 9 $\Delta x = \frac{9-1}{4} = 2$)

(3) $\int \frac{2x^2-11x-6}{x^3-2x^2}\,dx$

$\frac{2x^2-11x-6}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$

$2x^2-11x-6 = Ax(x-2) + B(x-2) + Cx^2$

$x=0$: $-6 = -2B$ $B = 3$

$x=2$: $-20 = 4C$ $C = -5$

x^2 COEFF.: $2 = A + C$ $A = 7$

$\int\left(\frac{7}{x} + \frac{3}{x^2} - \frac{5}{x-2}\right)dx = \boxed{7\ln|x| - \frac{3}{x} - 5\ln|x-2| + C}$

(4) $\int_1^3 x^2\ln x\,dx$ Let $u = \ln x$, $dv = x^2\,dx$

$du = \frac{1}{x}\,dx$, $v = \frac{x^3}{3}$

$= \left[\frac{x^3}{3}\ln x - \frac{1}{3}\int x^2\,dx\right]_1^3 = \left[\frac{x^3}{3}\ln x - \frac{1}{9}x^3\right]_1^3 = (9\ln 3 - 3) - \left(0 - \frac{1}{9}\right) = \boxed{9\ln 3 - \frac{26}{9}}$

(5) $\int_4^{12}\frac{24}{\sqrt{x}\,(x+4)}\,dx$ Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\,dx$ IF $x = 4$, $u = 2$; $x = 12$, $u = \sqrt{12} = 2\sqrt{3}$

$= 24\cdot 2\int_4^{12}\frac{1}{x+4}\cdot\frac{1}{2\sqrt{x}}\,dx = 48\int_2^{2\sqrt{3}}\frac{1}{u^2+4}\,du = 48\left[\frac{1}{2}\tan^{-1}\frac{u}{2}\right]_2^{2\sqrt{3}} = 24\left(\tan^{-1}\sqrt{3} - \tan^{-1}1\right)$

$= 24\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \boxed{2\pi}$

(6) $\int_1^\infty \frac{30x}{(x^2+2)^2}\,dx = \lim_{t\to\infty}\int_1^t \frac{30x}{(x^2+2)^2}\,dx$ Let $u = x^2+2$, $du = 2x\,dx$ IF $x = 1$, $u = 3$; $x = t$, $u = t^2+2$

$= \lim_{t\to\infty}\frac{30}{2}\int_1^t\frac{1}{(x^2+2)^2}\cdot 2x\,dx = \lim_{t\to\infty} 15\int_3^{t^2+2}\frac{1}{u^2}\,du = \lim_{t\to\infty} 15\left[-\frac{1}{u}\right]_3^{t^2+2}$

$= \lim_{t\to\infty} 15\left(-\frac{1}{t^2+2} - \left(-\frac{1}{3}\right)\right) = 15\left(0 + \frac{1}{3}\right) = \boxed{5}$

(7) A) $f(x) = \sqrt{x}$, $a = 100$

$f'(x) = \frac{1}{2} x^{-1/2}$

$f''(x) = -\frac{1}{4} x^{-3/2}$

$f'''(x) = \frac{3}{8} x^{-5/2}$

n	$f^{(n)}(a)$	$\frac{f^{(n)}(a)}{n!}$
0	10	10
1	1/20	1/20
2	−1/4,000	−1/8,000
3	3/800,000	1/1,600,000

$$P_3(x) = \boxed{10 + \frac{1}{20}(x-100) - \frac{1}{8{,}000}(x-100)^2 + \frac{1}{1{,}600{,}000}(x-100)^3}$$

B) $\sqrt{102} \approx P_3(102) = \boxed{10 + \frac{1}{20}(2) - \frac{1}{8{,}000}(4) + \frac{1}{1{,}600{,}000}(8)} = \underline{10.099505}$

(ACTUAL VALUE = 10.0995049 4 ...)

(8) $\int x^3 e^{x^2}\,dx$ LET $T = x^2$, $dT = 2x\,dx$

$= \frac{1}{2}\int x^2 e^{x^2} \cdot 2x\,dx = \frac{1}{2}\int T e^T\,dT$ LET $u = T$, $dv = e^T dT$; $du = dT$, $v = e^T$

$= \frac{1}{2}\left[T e^T - \int e^T dT\right] = \frac{1}{2}\left[T e^T - e^T\right] + C = \boxed{\frac{1}{2}\left[x^2 e^{x^2} - e^{x^2}\right] + C}$

OR LET $u = x^2$, $dv = x e^{x^2} dx$; $du = 2x\,dx$, $v = \frac{1}{2} e^{x^2}$ TO GET $v = \int x e^{x^2} dx$ LET $T = x^2$, $dT = 2x\,dx$

$= \frac{1}{2}\int e^{x^2} \cdot 2x\,dx = \frac{1}{2}\int e^T dT$

$= \frac{1}{2} e^T + C = \frac{1}{2} e^{x^2}$ (TAKING $C=0$)

$\frac{1}{2} x^2 e^{x^2} - \int x e^{x^2} dx = \boxed{\frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + C}$

(9) $\int \frac{x^2 + 16x + 18}{(x-1)(x^2+4)}\,dx$ $\qquad \frac{x^2+16x+18}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx + C}{x^2+4}$

$x^2 + 16x + 18 = A(x^2+4) + (Bx + C)(x-1)$

$x = 1$: $35 = 5A$ $\quad A = 7$

$x = 0$: $18 = 4A - C$ $\quad C = 10$

x^2 COEFF.: $1 = A + B$ $\quad B = -6$

$$\int \left(\frac{7}{x-1} + \frac{-6x+10}{x^2+4}\right) dx = 7\int \frac{1}{x-1}\,dx - 3\int \frac{2x}{x^2+4}\,dx + 10\int \frac{1}{x^2+4}\,dx$$

$$= \boxed{7 \ln|x-1| - 3\ln|x^2+4| + 10\left(\frac{1}{2}\tan^{-1}\frac{x}{2}\right) + C}$$

$$= \boxed{7 \ln|x-1| - 3\ln(x^2+4) + 5\tan^{-1}\frac{x}{2} + C}$$

(10) $\int \frac{x^2}{(x^2+16)^2}\,dx$ LET $u = x$, $dv = \frac{x}{(x^2+16)^2}\,dx$; $du = dx$, $v = \frac{1}{2}\left(-\frac{1}{x^2+16}\right)$

$v = \int \frac{x}{(x^2+16)^2}\,dx$ LET $T = x^2+16$, $dT = 2x\,dx$

$= \frac{1}{2}\int \frac{1}{T^2}\,dT = \frac{1}{2}\left(-\frac{1}{T}\right) + C$

$= \frac{1}{2}\left(-\frac{1}{x^2+16}\right)$ (IF $C=0$)

$$= x\left(-\frac{1}{2(x^2+16)}\right) - \left(-\frac{1}{2}\right)\int \frac{1}{x^2+16}\,dx$$

$$= \boxed{-\frac{x}{2(x^2+16)} + \frac{1}{2}\left(\frac{1}{4}\tan^{-1}\frac{x}{4}\right) + C} = \boxed{-\frac{x}{2(x^2+16)} + \frac{1}{8}\tan^{-1}\frac{x}{4} + C}$$

REMARK WE COULD ALSO USE $x = 4\tan\theta$, SO $dx = 4\sec^2\theta\,d\theta$ AND $x^2+16 = 16\sec^2\theta$

(11) $\frac{dY}{dT} = 2Y + 3$, $Y(0) = 4$

$\int \frac{1}{2Y+3}\,dY = \int dT$, $\frac{1}{2}\int \frac{2}{2Y+3}\,dY = T + C$, $\frac{1}{2}\ln(2Y+3) = T + C$,

$\ln(2Y+3) = 2T + D$, $2Y + 3 = e^{2T+D} = e^D \cdot e^{2T} = Ae^{2T}$, $2Y = Ae^{2T} - 3$,

$\underline{Y = \frac{1}{2}(Ae^{2T} - 3)}$ WHEN $T=0$, $Y=4$: $4 = \frac{1}{2}(A-3)$ SO $A - 3 = 8$, $\underline{A = 11}$

SO $\boxed{Y = \frac{1}{2}(11e^{2T} - 3)}$

OR 1) $\frac{dY}{dT} - 2Y = 3$ 2) $u(T) = e^{\int -2\,dT} = \underline{e^{-2T}}$

3) $e^{-2T}\left[\frac{dY}{dT} - 2Y\right] = 3e^{-2T}$, $(e^{-2T}Y)' = 3e^{-2T}$

4) $e^{-2T}Y = \int 3e^{-2T}\,dT = -\frac{3}{2}e^{-2T} + C$ SO $\underline{Y = -\frac{3}{2} + Ce^{2T}}$

WHEN $T=0$, $Y=4$: $4 = -\frac{3}{2} + C$ SO $\underline{C = \frac{11}{2}}$: $\boxed{Y = -\frac{3}{2} + \frac{11}{2}e^{2T}}$

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(12) $\int \frac{5x-7}{x^2-6x+34}\,dx = \frac{5}{2}\int \frac{2x-6}{x^2-6x+34}\,dx + \int \frac{8}{x^2-6x+34}\,dx$

$= \frac{5}{2}\ln|x^2-6x+34| + 8\int \frac{1}{(x-3)^2+25}\,dx$ ← LET $\underline{u = x-3}$, $du = dx$

$= \frac{5}{2}\ln|x^2-6x+34| + 8\left(\frac{1}{5}\tan^{-1}\frac{u}{5}\right) + C$

$= \boxed{\frac{5}{2}\ln(x^2-6x+34) + \frac{8}{5}\tan^{-1}\left(\frac{x-3}{5}\right) + C}$

(SINCE $x^2 - 6x + 34 > 0$ FOR ALL x, THE ABSOLUTE VALUES ARE NOT NEEDED.)

OR $\int \frac{5x-7}{x^2-6x+34}\,dx = \int \frac{5x-7}{(x-3)^2+25}\,dx$ LET $\underline{u = x-3}$, $x = u+3$, $dx = du$

$= \int \frac{5(u+3)-7}{u^2+25}\,du = \int \frac{5u+8}{u^2+25}\,du = \int \frac{5u}{u^2+25}\,du + \int \frac{8}{u^2+25}\,du$

$= \frac{5}{2}\int \frac{2u}{u^2+25}\,du + 8\int \frac{1}{u^2+25}\,du$

$= \frac{5}{2}\ln(u^2+25) + 8\left(\frac{1}{5}\tan^{-1}\frac{u}{5}\right) + C$

$= \boxed{\frac{5}{2}\ln((x-3)^2+25) + \frac{8}{5}\tan^{-1}\left(\frac{x-3}{5}\right) + C}$

$= \boxed{\frac{5}{2}\ln(x^2-6x+34) + \frac{8}{5}\tan^{-1}\left(\frac{x-3}{5}\right) + C}$