

Math 17B  
Test 3

Printed Name \_\_\_\_\_

(FIRST)

(LAST)

Signature \_\_\_\_\_

**Please Show All Your Work, and Mark Your Answers Clearly.**

**No Calculators -- No Scratch Paper -- No Cell Phones**

There are **4 pages** of problems. (The last problem is for extra credit.)

**You are expected to do your own work, and to adhere to the UCD Code of Academic Conduct.**

**Simplify all numerical answers.**

Please indicate clearly if you continue work on the back of a page.

Please stop working **immediately** when time is called;  
**you are subject to a deduction from your test score** if you do not.

- ① For the DE  $\frac{dy}{dt} = 5(y-1)(y-4)(y-7)^2$ , use the graphical approach to classify each equilibrium as stable, unstable, or semistable.

P.1

7  
pts

- ② Use matrix reduction to solve the linear system

$$x + 2y + 2z + 4w = 3$$

$$2x + 3y + z + 2w = 7.$$

9  
pts

- ③ Find the solution of the DE  $xy' - 2y = 8x^4 - 5x^2$  whose graph passes through  $(1, 7)$ .

12  
pts

④ FIND THE SOLUTION OF THE DE  $\frac{dy}{dt} = t^2(y^2 + 9)$  SATISFYING  $y(0) = 3$ .

P. 2

9  
PTS

⑤ A TANK INITIALLY CONTAINS 200 GALLONS OF WATER WITH 1.5 LB OF DYE PER GALLON. IF WATER CONTAINING 4 LB OF DYE PER GALLON ENTERS THE TANK AT THE RATE OF 2 GAL/MIN, AND THE STIRRED MIXTURE IS DRAINED AT THE RATE OF 5 GAL/MIN, SET UP A DE FOR  $A(t)$ , THE AMOUNT OF DYE IN THE TANK AT TIME  $t$ .

8  
PTS

⑥ A HERD OF 100 BUFFALO IS MOVED TO A REGION WHICH CAN SUSTAIN A MAXIMUM OF 600 BUFFALO. IF THE POPULATION INCREASES ACCORDING TO THE LOGISTIC GROWTH MODEL, AND IF THERE ARE 200 BUFFALO AFTER 15 YEARS, FIND WHEN THERE WILL BE 400 BUFFALO.

12  
PTS

⑦ IF  $A = \begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}$  AND  $B^{-1} = \begin{bmatrix} 5 & 4 \\ 6 & 8 \end{bmatrix}$ , FIND  $(ABT)^{-1}$ .

P.3

10  
PTS

⑧ IF  $A = \begin{bmatrix} 4 & 1 \\ 8 & -3 \end{bmatrix}$ , FIND THE EIGENVALUES AND CORRESPONDING EIGENVECTORS FOR A.

12  
PTS

⑨ A POPULATION IS DIVIDED INTO 3 AGE CLASSES, AND FEMALES OF AGES 0, 1, AND 2 HAVE AN AVERAGE OF 1.5, 5.0, AND 3.2 FEMALE OFFSPRING, RESPECTIVELY. IF 50% OF FEMALES AGE 0 AND 80% OF FEMALES AGE 1 SURVIVE UNTIL THE END OF THE NEXT BREEDING SEASON, FIND THE AGE DISTRIBUTION AT TIME 1 IF INITIALLY THERE ARE 40, 30, AND 50 FEMALES OF AGES 0, 1, AND 2, RESPECTIVELY.

8  
PTS

10 Solve the DE  $\frac{dy}{dx} = 8x^3y - 2x^3y^2$  for  $y$ .

P.4

13  
PTS

11 OIL IS LEAKING INTO A LAKE AT A CONTINUOUS RATE OF 5 GAL/DAY, AND IT IS REMOVED FROM THE LAKE AT A RATE PROPORTIONAL TO THE AMOUNT PRESENT. THERE WAS 10 GAL OF OIL IN THE LAKE INITIALLY, AND THE AMOUNT OF OIL HAS A LIMITING VALUE OF 100 GALLONS AS TIME INCREASES. FIND A FORMULA FOR  $Y(T)$ , THE AMOUNT OF OIL IN THE LAKE AFTER  $T$  DAYS.

12  
PTS  
(EXTRA  
CREDIT)