(43) $\int_0^3 x\sqrt{x^2+1}\,dx$ LET $\underline{u = x^2+1}$, $du = 2x\,dx$ IF $x=0,\ u=1$; $x=3,\ u=10$

$= \frac{1}{2}\int_0^3 \sqrt{x^2+1}\cdot 2x\,dx = \frac{1}{2}\int_1^{10}\sqrt{u}\,du = \frac{1}{2}\int_1^{10} u^{1/2}\,du = \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_1^{10} = \boxed{\frac{1}{3}\left(10^{3/2}-1\right)}$

(48) $\int_{\ln 4}^{\ln 7} \frac{e^x}{(e^x-3)^2}\,dx$ LET $\underline{u = e^x-3}$, $du = e^x\,dx$ IF $x=\ln 4,\ u=e^{\ln 4}-3 = 4-3 = 1$; $x = \ln 7,\ u = e^{\ln 7}-3 = 7-3 = 4$

$= \int_{\ln 4}^{\ln 7}\frac{1}{(e^x-3)^2}\cdot e^x\,dx = \int_1^4 \frac{1}{u^2}\,du = \int_1^4 u^{-2}\,du = \left[-\frac{1}{u}\right]_1^4 = -\frac{1}{4}-(-1) = \boxed{\frac{3}{4}}$

(49) $\int_0^{\pi/3}\sin x\cos x\,dx$ LET $\underline{u = \sin x}$, $du = \cos x\,dx$ IF $x=0,\ u=\sin 0 = 0$; $x=\frac{\pi}{3},\ u = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$= \int_0^{\sqrt{3}/2} u\,du = \left[\frac{1}{2}u^2\right]_0^{\sqrt{3}/2} = \frac{1}{2}\left(\frac{3}{4}-0\right) = \boxed{\frac{3}{8}}$

OR $\int_0^{\pi/3}\sin x\cos x\,dx = \frac{1}{2}\int_0^{\pi/3}\underline{2\sin x\cos x}\,dx = \frac{1}{2}\int_0^{\pi/3}\underline{\sin 2x}\,dx = \frac{1}{2}\left[-\frac{1}{2}\cos 2x\right]_0^{\pi/3}$

$= -\frac{1}{4}\left(\cos\frac{2\pi}{3}-\cos 0\right) = -\frac{1}{4}\left(-\frac{1}{2}-1\right) = -\frac{1}{4}\left(-\frac{3}{2}\right) = \boxed{\frac{3}{8}}$

(51) $\int_0^{\pi/4}\tan x\sec^2 x\,dx$ LET $\underline{u = \tan x}$, $du = \sec^2 x\,dx$ IF $x=0,\ u=\tan 0 = 0$; $x=\pi/4,\ u=\tan\frac{\pi}{4} = 1$

$= \int_0^1 u\,du = \left[\frac{1}{2}u^2\right]_0^1 = \frac{1}{2}(1-0) = \boxed{\frac{1}{2}}$

REMARK In #49, we could also let $u = \cos x$; and in #51, we could also let $u = \sec x$.

(52) $\int_0^{\pi/3}\frac{\sin x}{\cos^2 x}\,dx = \int_0^{\pi/3}\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\,dx = \int_0^{\pi/3}\tan x\sec x\,dx = \left[\sec x\right]_0^{\pi/3} = \left(\sec\frac{\pi}{3}-\sec 0\right)$

$= \frac{1}{1/2}-\frac{1}{1} = 2-1 = \boxed{1}$

OR LET $\underline{u = \cos x}$, $u = -\sin x\,dx$ IF $x=0,\ u=\cos 0 = 1$; $x=\pi/3,\ u = \cos\frac{\pi}{3} = \frac{1}{2}$

$-\int_0^{\pi/3}\frac{1}{\cos^2 x}(-\sin x)\,dx = -\int_1^{1/2}\frac{1}{u^2}\,du = \int_{1/2}^1 u^{-2}\,du = \left[-\frac{1}{u}\right]_{1/2}^1 = -\left(1-\frac{1}{1/2}\right) = -1+2 = \boxed{1}$

(53) $\int_5^9\frac{x}{x-3}\,dx = \int_5^9\left(1+\frac{3}{x-3}\right)dx \longleftarrow \frac{x}{x-3} = \frac{(x-3)+3}{x-3} = \frac{x-3}{x-3}+\frac{3}{x-3} = 1+\frac{3}{x-3}$

$= \left[x+3\ln(x-3)\right]_5^9 = (9+3\ln 6)-(5+3\ln 2) = 4+3(\ln 6-\ln 2) = \boxed{4+3\ln 3}$

OR LET $\underline{u = x-3}$, $x = u+3$, $dx = du$ IF $x=5,\ u=2$; $x=9,\ u=6$

$\int_5^9\frac{x}{x-3}\,dx = \int_2^6\frac{u+3}{u}\,du = \int_2^6\left(1+\frac{3}{u}\right)du = \left[u+3\ln u\right]_2^6$

$= (6+3\ln 6)-(2+3\ln 2) = 4+3(\ln 6-\ln 2) = \boxed{4+3\ln 3}$

MATH 17B SEC. 7.1 AND 7.2 — SOLUTIONS

7.1 – (55) $\displaystyle\int_e^{e^2} \frac{dx}{x(\ln x)^2}$ LET $u=\ln x$, $du=\frac{1}{x}\,dx$ IF $x=e$, $u=\ln e=1$; $x=e^2$, $u=\ln e^2=2$

$$= \int_e^{e^2}\frac{1}{(\ln x)^2}\cdot\frac{1}{x}\,dx = \int_1^2 \frac{1}{u^2}\,du = \int_1^2 u^{-2}\,du = \left[-\frac{1}{u}\right]_1^2 = -\frac{1}{2}-(-1) = \boxed{\frac{1}{2}}$$

(57) $\displaystyle\int_1^9 \frac{1}{\sqrt{x}}\, e^{-\sqrt{x}}\,dx$ LET $u=-\sqrt{x}$, $du = -\frac{1}{2\sqrt{x}}\,dx$ IF $x=1$, $u=-1$; $x=9$, $u=-3$

$$=(-2)\int_1^9 e^{-\sqrt{x}}\cdot\left(-\frac{1}{2}\right)\cdot\frac{1}{\sqrt{x}}\,dx = -2\int_{-1}^{-3} e^u\,du = 2\int_{-3}^{-1} e^u\,du = 2\left[e^u\right]_{-3}^{-1} = \boxed{2\left(e^{-1}-e^{-3}\right)}$$

(58) $\displaystyle\int_0^2 x\sqrt{4-x^2}\,dx$ LET $u=4-x^2$, $du=-2x\,dx$ IF $x=0$, $u=4$; $x=2$, $u=0$

$$=\left(-\frac{1}{2}\right)\int_0^2 \sqrt{4-x^2}\,(-2)x\,dx = -\frac{1}{2}\int_4^0 \sqrt{u}\,du = \frac{1}{2}\int_0^4 u^{1/2}\,du = \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_0^4 = \frac{1}{3}\left(4^{3/2}-0\right)$$
$$= \frac{1}{3}\left(2^3\right) = \boxed{\frac{8}{3}}$$

7.2 – (1) $\displaystyle\int x\cos x\,dx$ LET $u=x$, $dv=\cos x\,dx$; $du=dx$, $v=\sin x$

$$= x\sin x - \int \sin x\,dx = x\sin x-(-\cos x)+C = \boxed{x\sin x+\cos x+C}$$

(7) $\displaystyle\int x e^x\,dx$ LET $u=x$, $dv=e^x\,dx$; $du=dx$, $v=e^x$

$$= xe^x - \int e^x\,dx = \boxed{xe^x-e^x+C}$$

(9) $\displaystyle\int x^2 e^x\,dx$ LET $u=x^2$, $dv=e^x\,dx$; $du=2x\,dx$, $v=e^x$

$$= x^2e^x - 2\int xe^x\,dx \quad \longleftarrow \text{(NOW USE \#7)}$$
$$= x^2e^x - 2\left[xe^x-e^x\right]+C = \boxed{x^2e^x-2xe^x+2e^x+C}$$

OR

u		dv
x^2	$+$	$e^x\,dx$
$2x$	$-$	e^x
2	$+$	e^x
0		e^x

$$\int x^2e^x\,dx = \boxed{x^2e^x-2xe^x+2e^x+C}$$

(13) $\displaystyle\int x\ln 3x\,dx$ LET $u=\ln 3x$, $dv=x\,dx$; $du=\frac{\cancel{3}}{\cancel{3}x}\,dx$, $v=\frac{x^2}{2}$

$$= \frac{x^2}{2}\ln 3x - \frac{1}{2}\int x^2\cdot\frac{1}{x}\,dx$$
$$= \frac{x^2}{2}\ln 3x - \frac{1}{2}\int x\,dx = \boxed{\frac{x^2}{2}\ln 3x-\frac{1}{2}\cdot\frac{x^2}{2}+C} = \boxed{\frac{1}{2}x^2\ln 3x-\frac{1}{4}x^2+C}$$