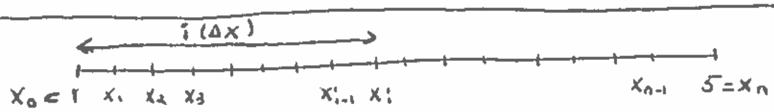
Find $\int_1^5 (3x^2 - 10x + 9)\,dx$ using the def. of the definite integral as a limit of Riemann sums, using partitions of $[1,5]$ into n equal subintervals and right endpoints as sampling numbers.



$$\Delta x = \frac{5-1}{n} = \frac{4}{n}$$

$$c_i = x_i = a + i(\Delta x) = 1 + i\left(\frac{4}{n}\right)$$

$$\int_1^5 (3x^2 - 10x + 9)\,dx = \lim_{\|P\| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

$$= \lim_{\|P\| \to 0} \sum_{i=1}^{n} (3c_i^2 - 10c_i + 9)\Delta x$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left(3\left(1 + \frac{4i}{n}\right)^2 - 10\left(1 + \frac{4i}{n}\right) + 9\right) \cdot \frac{4}{n}$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left(3\left(1 + \frac{8i}{n} + \frac{16i^2}{n^2}\right) - 10\left(1 + \frac{4i}{n}\right) + 9\right) \cdot \frac{4}{n}$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left(2 - \frac{16i}{n} + \frac{48i^2}{n^2}\right) \cdot \frac{4}{n}$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{8}{n} - \frac{64i}{n^2} + \frac{192i^2}{n^3}\right)$$

$$= \lim_{n \to \infty} \left[\sum_{i=1}^{n} \frac{8}{n} - \frac{64}{n^2}\sum_{i=1}^{n} i + \frac{192}{n^3}\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n \to \infty} \left[\frac{8}{n}(n) - \frac{64}{n^2} \cdot \frac{n(n+1)}{2} + \frac{192}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n \to \infty} \left[8 - 32 \cdot \frac{n+1}{n} + 32 \cdot \frac{2n^2 + 3n + 1}{n^2}\right]$$

$$= 8 - 32 \cdot 1 + 32 \cdot 2 = \boxed{40}$$

Find $\int_a^b x^2\,dx$ for $0<a<b$ using the def. of the definite integral as a limit of Riemann sums, using arbitrary partitions of $[a,b]$ and sampling numbers given by

$$c_i = \sqrt{\frac{x_{i-1}^2 + x_{i-1}x_i + x_i^2}{3}} \quad \text{for } 1 \le i \le n.$$

$$\int_a^b x^2\,dx = \lim_{\|P\|\to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i$$

$$= \lim_{\|P\|\to 0} \sum_{i=1}^{n} c_i^2\,\Delta x_i$$

$$= \lim_{\|P\|\to 0} \sum_{i=1}^{n} \left(\frac{x_{i-1}^2 + x_{i-1}x_i + x_i^2}{3}\right)(x_i - x_{i-1})$$

$$= \lim_{\|P\|\to 0} \frac{1}{3}\sum_{i=1}^{n} (x_i - x_{i-1})(x_i^2 + x_i x_{i-1} + x_{i-1}^2)$$

$$= \lim_{\|P\|\to 0} \frac{1}{3}\sum_{i=1}^{n} (x_i^3 - x_{i-1}^3)$$

$$= \lim_{\|P\|\to 0} \frac{1}{3}\left[(\cancel{x_1^3} - x_0^3) + (\cancel{x_2^3} - \cancel{x_1^3}) + (\cancel{x_3^3} - \cancel{x_2^3}) + \cdots + (x_n^3 - \cancel{x_{n-1}^3})\right]$$

$$= \lim_{\|P\|\to 0} \frac{1}{3}\left[x_n^3 - x_0^3\right]$$

$$= \lim_{\|P\|\to 0} \frac{1}{3}(b^3 - a^3)$$

$$= \boxed{\frac{1}{3}(b^3 - a^3)}$$
