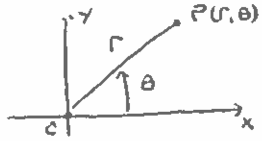
MATH 17B　　　　POLAR COORDINATES (sec. 9.3)

DEF The point P with polar coordinates (r, θ) is

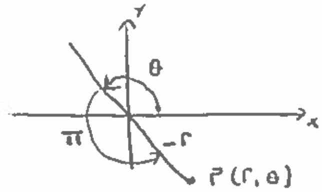
1) r units from the origin along the terminal side of θ, if $r > 0$.

2) $-r$ units from the origin along the terminal side of $\theta + \pi$, if $r < 0$.

$\boxed{r > 0}$



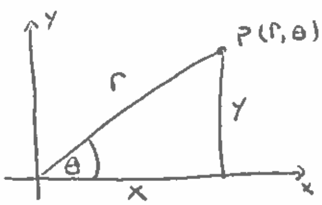
$\boxed{r < 0}$



REMARK Notice that polar coordinates are not unique!

The point with polar coordinates (r, θ) also has polar coordinates $(r, \theta + 2n\pi)$ and $(-r, \theta + (2n+1)\pi)$ for any integer n.

RELATIONS BETWEEN POLAR AND RECTANGULAR COORDINATES



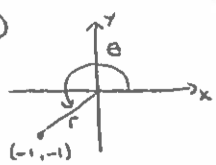
1) $\cos\theta = \frac{x}{r}$, so $\boxed{x = r\cos\theta}$

2) $\sin\theta = \frac{y}{r}$, so $\boxed{y = r\sin\theta}$

3) $\boxed{r^2 = x^2 + y^2}$ by the Pythagorean Theorem

4) $\tan\theta = \frac{y}{x}$

Ex ⑥

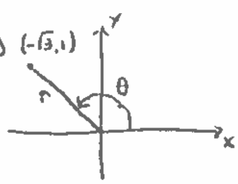


$X = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$r^2 = x^2 + y^2 = 1 + 1 = 2$, so $\boxed{r = \sqrt{2}}$ (taking $r > 0$)

$\tan\theta = \frac{y}{x} = \frac{-1}{-1} = 1$, so $\boxed{\theta = \frac{5\pi}{4}} \longleftarrow \frac{\pi}{4} + \pi$, since θ is in Q. III

⑦



$X = \begin{bmatrix} -\sqrt{3} \\ 1 \end{bmatrix}$

$r^2 = x^2 + y^2 = 3 + 1 = 4$, so $\boxed{r = 2}$ (taking $r > 0$)

$\tan\theta = \frac{y}{x} = \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}}$, so $\boxed{\theta = \frac{5\pi}{6}} \longleftarrow \pi - \frac{\pi}{6}$, since θ is in Q. II

⑨ $r = 2$, $\theta = 30°$

$x = r\cos\theta = 2\cos 30° = 2\left(\frac{\sqrt{3}}{2}\right) = \boxed{\sqrt{3}}$

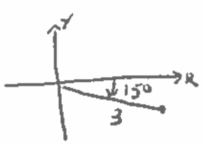
$y = r\sin\theta = 2\sin 30° = 2\left(\frac{1}{2}\right) = \boxed{1}$

⑩ $r = 3$, $\theta = 150°$

$x = r\cos\theta = 3\cos 150° = 3\left(-\frac{\sqrt{3}}{2}\right) = \boxed{-\frac{3\sqrt{3}}{2}}$

$y = r\sin\theta = 3\sin 150° = 3\left(\frac{1}{2}\right) = \boxed{\frac{3}{2}}$

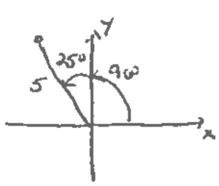
⑬



$x = r\cos\theta = 3\cos(-15°) = \boxed{3\cos 15°} = \frac{3}{4}(\sqrt{6} + \sqrt{2})$

$y = r\sin\theta = 3\sin(-15°) = \boxed{-3\sin 15°} = \frac{3}{4}(\sqrt{2} - \sqrt{6})$

⑮



$x = r\cos\theta = \boxed{5\cos 115°} = -5\cos 65°$

$y = r\sin\theta = \boxed{5\sin 115°} = 5\sin 65°$