MATH 17B   Sec. 8.1 - FIRST-ORDER LINEAR DE'S

DEF A FIRST-ORDER LINEAR DE IS A DE WHICH CAN BE WRITTEN IN THE FORM

$$\boxed{\frac{dy}{dx} + P(x)y = Q(x)}.$$

PROCEDURE TO SOLVE FIRST-ORDER LINEAR DE'S

1) WRITE THE DE IN THE STANDARD FORM ABOVE.

2) FIND $\boxed{u(x) = e^{\int P(x)\,dx}}$, WHICH IS CALLED THE INTEGRATING FACTOR FOR THE DE.

3) MULTIPLY BOTH SIDES OF THE DE BY $u(x)$ TO GET

$$u(x)\frac{dy}{dx} + u(x)P(x)y = u(x)Q(x)$$

$$u(x)y' + u'(x)y = u(x)Q(x) \qquad \left[\text{SINCE } u'(x) = e^{\int P(x)dx}\cdot D_x\left(\int P(x)dx\right) = u(x)P(x)\right]$$

$$\frac{d}{dx}(u(x)y) = u(x)Q(x) \qquad \left[\text{SINCE } \frac{d}{dx}(u(x)y) = u(x)y' + u'(x)y\right]$$

4) FIND $u(x)y = \int u(x)Q(x)\,dx$, AND THEN SOLVE FOR y.

EX A) SOLVE $y' = e^x - 3y$.

1) $y' + 3y = e^x$

2) $u(x) = e^{\int 3\,dx} = e^{3x}$

3) $e^{3x}[y' + 3y] = e^{3x}\cdot e^x$

$$\frac{d}{dx}(e^{3x}y) = e^{4x}$$

4) $e^{3x}y = \int e^{4x}\,dx = \frac{1}{4}e^{4x} + C$ (NOW DIVIDE BY e^{3x}, OR EQUIVALENTLY MULTIPLY BY e^{-3x})

$$\boxed{y = \frac{1}{4}e^x + Ce^{-3x}}$$

B) SOLVE $xy' - 2y = 6x^4 + 4x^2$, $x > 0$.

1) $y' - \frac{2}{x}y = 6x^3 + 4x$

2) $u(x) = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = (e^{\ln x})^{-2} = x^{-2}$

3) $x^{-2}\left[y' - \frac{2}{x}y\right] = x^{-2}\left[6x^3 + 4x\right]$

$$\frac{d}{dx}(x^{-2}y) = 6x + 4x^{-1}$$

4) $x^{-2}y = \int(6x + 4x^{-1})\,dx = 3x^2 + 4\ln x + C$ (SINCE $x > 0$) (NOW MULTIPLY BY x^2)

$$\boxed{y = 3x^4 + 4x^2\ln x + Cx^2}$$