1. \((1 - i)^2 = (1 - i)(1 - i) = 1 - 2i + i^2 = 1 - 2i - 1 = -2i\)

2. \(\log ti = \log (n + i) = \left( \frac{\pi}{2} + 2\pi n \right), \ n \in \mathbb{Z}\)

3. \(A = C - \{ \frac{1}{2}, -3 \}\)

   a) \(f(z)\) is analytic on \(A\), since it is analytic on all of \(C\).

   b) \(g(z) = \frac{1}{\sin z}\) is undefined when \(\sin z = 0\), so when \(z = n\pi\) for any \(n \in \mathbb{Z}\).

   c) \(h(z) = \frac{\cos 2\pi z}{z^2 - 1}\) is analytic on \(A\), since it is analytic everywhere except at \(z = \frac{1}{2}\), \(z = -1\).

4. \(\left| \frac{a - b}{1 - ab} \right| < 1 \iff 1 - |a - b| < |1 - ab| \iff |a - b|^2 < |1 - ab|^2 \iff (a - b)(\overline{a - b}) < (1 - ab)(1 - \overline{ab}) \iff a\overline{a} - a\overline{b} - b\overline{a} + \overline{b}b < 1 - ab - a\overline{b} + a\overline{a}b\overline{b}\)

   \(\iff |a|^2 + |b|^2 < 1 + |a|^2|b|^2 \iff 1 + |a|^2|b|^2 - |a|^4 - |b|^4 > 0 \iff (1 - |a|^2)(1 - |b|^2) > 0\), and this follows from the assumption that \(|a| < 1, |b| < 1\).

5. \(u(x, y) = e^{xy} \cos x\)

   \(u_x = -e^{xy} \sin x\) and \(u_y = e^{xy} \cos x\), so

   \(u_{xx} + u_{yy} = -e^{xy} \cos x + e^{xy} \cos x = (c^2 - 1)e^{xy} \cos x = 0\) for \(c^2 = 1\), so \(c = \pm 1\) (since \(u\) must be a harmonic function).

   1. \(v_x = -e^{xy} \sin x\) for \(c = 1\),

   \(v_y = e^{xy} \cos x + g(x)\), and \(v_x = -u_y = -e^{xy} \cos x\) gives \(g'(x) = 0\) so \(g(x) = k\) for some \(k \in \mathbb{R}\).

   Therefore \(f = u + iv = e^{xy} \cos x + (e^{xy} \sin x + k), \) so \(f(z) = e^{ix} + ik, \ k \in \mathbb{R}\)

   b) \(v_x = -e^{-xy} \sin x\) for \(c = 1\),

   \(v_y = e^{-xy} \cos x + g(x)\), and \(v_x = -u_y = e^{-xy} \cos x\) gives \(g'(x) = 0\) so \(g(x) = k\) for some \(k \in \mathbb{R}\).

   Therefore \(f = u + iv = e^{xy} \sin x + (e^{xy} \sin x + k), \) so \(f(z) = e^{ix} + iK, \ k \in \mathbb{R}\)

   Using \(e^{-i(x+iy)} = e^{y(-i-x)} = e^{-y}e^{-ix} = e^{-y}e^y \cos(-ix) = e^{-y} \cos x - i e^{-y} \sin x\)

   and \(e^{ix} = e^{i(x+iy)} = e^{-y}e^{iy} = e^{-y} \cos x + i e^{-y} \sin x\)
1) The line segment from 0 to 1 maps to the line segment from 0 to 1.

2) The line segment from 1 to 1 + i maps to a segment of the parabola \( u = 1 - \frac{v^2}{4} \)

3) From 1 to \( 1 + i \), \( z = i + t \), \( 0 \leq t \leq 1 \), gives \( z = i(1 - t^2) + 2ti \)

4) So \( u = 1 - t^4 \) and \( v = 2t \).

5) The line segment from 0 to 1 + i maps to the line segment from 0 to \( 1 + i \).

6) Since \( \frac{d}{dz} (\sqrt{z}) = 2 \sqrt{z} \neq 0 \), the map is conformal at \( 1 \) and at \( 1 + i \), so it preserves the angles at these points. (The angle at 0, though, is doubled.)

\[ a) \quad \oint_{\gamma} \overline{z} \, dz = \int_{0}^{\pi} e^{it} \int_{0}^{\pi} (-i + e^{-it}) \int_{0}^{\pi} e^{it} \, dt = \int_{0}^{\pi} e^{it} + i \, dt = \left[ \frac{1}{i} e^{it} + i t \right]_{0}^{\pi} = 0 \]

\[ b) \quad \oint_{\gamma} \frac{1}{z - \lambda} \, dz = 0 \]