1) **Arc Length**

If a curve has parametric equations $x = f(t), y = g(t)$; $a \leq t \leq b$ (where $f'$ and $g'$ are continuous on $[a,b]$ and never both 0), its arc length is given by:

$$L = \int_{a}^{b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

**Remark** If we approximate each small piece of the curve by a line segment,

$$ds \approx \sqrt{(dx)^2 + (dy)^2}, \quad \Delta t \approx \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}, \quad \Delta$$

**Special Cases**

1. For the curve $y = f(x)$, replacing $t$ by $x$ gives:

$$L = \int_{a}^{b} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

(Use $x = t$, $y = f(t)$)

2. For the curve $x = f(y)$, replacing $t$ by $y$ gives:

$$L = \int_{a}^{b} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy$$

(Use $y = t$, $x = f(t)$)

II) **Area of a Surface of Revolution**

If a curve has parametric equations $x = f(t), y = g(t)$; $a \leq t \leq b$ (where $f'$ and $g'$ are continuous on $[a,b]$ and never both 0), the area of the surface generated by revolving the curve about a line is given by:

$$S = \int_{a}^{b} 2\pi r(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

where $r(t)$ is the distance from a typical point $P(t)$ on the curve to the line.

**Remark** If we approximate each small piece of the curve by a line segment,

$$A \approx 2\pi r \cdot \Delta L = 2\pi r(t) \sqrt{(dx)^2 + (dy)^2} \cdot \Delta t$$

**Special Cases**

1. For a curve $y = f(x)$, replacing $t$ by $x$ gives:

$$S = \int_{a}^{b} 2\pi f(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

(Use $x = t$, $y = f(t)$)

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(Use $y = t$, $x = f(t)$)