MATH 21C DISCUSSION SHEET (Sec. 14.4 – 14.5)

① Let $z = f\left(\frac{xy}{x+y}\right)$, where f is differentiable.

Find and simplify $x^2 \frac{\partial z}{\partial x} - y^2 \frac{\partial z}{\partial y}$.

② Let $z = x^2 f(x^2 - y^2)$, where f is differentiable.

Find and simplify $\frac{x z_y + y z_x}{z}$.

③ If $f(t) = \int_{t^2}^{t^5} e^{-t^4 x^2}\, dx$, find $f'(t)$ using the Chain Rule.

④ Let $f_x(3,7) = -6$ and $f_y(3,7) = 8$

a) Find the maximal directional derivative of f at $(3,7)$.

b) Find the directional derivative of f at $(3,7)$ in the direction of $\vec{v} = \vec{i} + 2\vec{j}$.

⑤ Find the directional derivative of $f(x,y,z) = x^3 y^2 z$ at $(1,2,5)$ in the direction of $\vec{v} = \vec{i} - 2\vec{j} - 2\vec{k}$.

⑥ Find the maximal directional derivative of $f(x,y,z) = \frac{x^2}{y} - yz$ at $P(4,2,1)$, and the direction in which it occurs.

⑦ If $z = f(x,y)$ where $x = s + t$ and $y = s - t$, use the Chain Rule to show that $\left(\frac{\partial z}{\partial s}\right)\left(\frac{\partial z}{\partial t}\right) = \left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2$.

⑧ Let $z = f(u,v)$, where $u = \frac{y}{x}$, $v = \frac{12x^2}{y}$, and f is differentiable.

If $f_{uu} = 2$, $f_{uv} = -1$, $f_{vv} = 5$, $f_u = -2$, and $f_v = 3$ at $(u,v) = (4,6)$,

find z_{xx} at $(x,y) = (2,8)$.