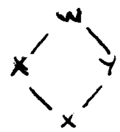
MATH 21C IMPLICIT DIFFERENTIATION AND THE CHAIN RULE (PP. 797–799)

WE CAN USE THE CHAIN RULE TO GET AN ALTERNATE WAY TO DIFFERENTIATE IMPLICITLY!

I) ASSUME THAT THE EQUATION $F(x,y)=C$ DEFINES y IMPLICITLY AS A DIFF. FUNCTION OF x, WHERE F IS DIFFERENTIABLE. IF WE LET $w=F(x,y)$, THEN WE GET



WHERE w IS CONSTANT AS A FUNCTION OF x (SINCE $w=F(x,g(x))=C$ IF $y=g(x)$).

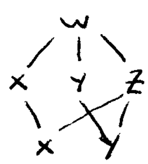
THEN $\dfrac{dw}{dx} = \dfrac{\partial w}{\partial x}\cdot\dfrac{dx}{dx} + \dfrac{\partial w}{\partial y}\cdot\dfrac{dy}{dx}$ BY THE CHAIN RULE,

SO $0 = F_x \cdot 1 + F_y \cdot \dfrac{dy}{dx}$ AND $\boxed{\dfrac{dy}{dx} = -\dfrac{F_x}{F_y}}$

Ex FIND $\dfrac{dy}{dx}$ IF $x^3+y^3-9xy=0$. (SEE EX. 5, P. 173)

LET $F(x,y)=x^3+y^3-9xy$, SO $\dfrac{dy}{dx} = -\dfrac{F_x}{F_y} = -\dfrac{3x^2-9y}{3y^2-9x} = \dfrac{\cancel{3}(3y-x^2)}{\cancel{3}(y^2-3x)} = \boxed{\dfrac{3y-x^2}{y^2-3x}}$

II) ASSUME THAT THE EQUATION $F(x,y,z)=C$ DEFINES z IMPLICITLY AS A DIFF. FUNCTION OF x AND y, WHERE F IS DIFF. IF WE LET $w=F(x,y,z)$, THEN WE GET



WHERE w IS CONSTANT AS A FUNCTION OF x AND y (SINCE $w=F(x,y,g(x,y))=C$ IF $z=g(x,y)$).

1) THEN $\left(\dfrac{\partial w}{\partial x}\right)_y = \left(\dfrac{\partial w}{\partial x}\right)_{y,z}\cdot\dfrac{dx}{dx} + \left(\dfrac{\partial w}{\partial z}\right)_{x,y}\cdot\dfrac{\partial z}{\partial x}$

SO $0 = F_x\cdot 1 + F_z\cdot\dfrac{\partial z}{\partial x}$ AND $\boxed{\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z}}$

SIMILARLY, 2) $\left(\dfrac{\partial w}{\partial y}\right)_x = \left(\dfrac{\partial w}{\partial y}\right)_{x,z}\cdot\dfrac{dy}{dy} + \left(\dfrac{\partial w}{\partial z}\right)_{x,y}\cdot\dfrac{\partial z}{\partial y}$

SO $0 = F_y\cdot 1 + F_z\cdot\dfrac{\partial z}{\partial y}$ AND $\boxed{\dfrac{\partial z}{\partial y} = -\dfrac{F_y}{F_z}}$

REMARK AS ON P. 844,

$\left(\dfrac{\partial w}{\partial x}\right)_y$ IS THE PARTIAL OF w WITH RESPECT TO x, TREATING w AS A FUNCTION OF x AND y

AND $\left(\dfrac{\partial w}{\partial x}\right)_{y,z}$ IS THE PARTIAL OF w WITH RESPECT TO x, TREATING w AS A FUNCTION OF x, y, AND z.

Ex FIND $\dfrac{\partial z}{\partial x}$ AT $(1,1,1)$ IF $xy+z^3x-2yz=0$. (SEE 14.3, #65)

LET $F(x,y,z)=xy+z^3x-2yz$; THEN

$\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z} = -\dfrac{y+z^3}{3z^2x-2y}$, SO $\left.\dfrac{\partial z}{\partial x}\right|_{(1,1,1)} = -\dfrac{2}{1} = \boxed{-2}$