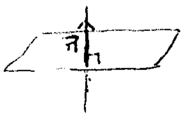
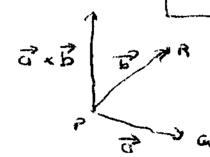
SOLUTIONS - SEC. 12.5

(3) $P(-2,0,3)$, $Q(3,5,-2)$. $\vec{a} = \overrightarrow{PQ} = \langle 5,5,-5\rangle$ and $\frac{1}{5}\vec{a} = \langle 1,1,-1\rangle$ are parallel to the line,

so the line is given by $\boxed{\begin{aligned} x &= -2+t \\ y &= t \\ z &= 3-t \end{aligned}}$ $\left(\text{or by } \boxed{\begin{aligned} x &= 3+t \\ y &= 5+t \\ z &= -2-t \end{aligned}}\right)$

(9) The line is perpendicular to the plane $x+2y+2z=13$, so the normal vector $\vec{n} = \langle 1,2,2\rangle$ for the plane is a direction vector for the line. Since it passes through $(0,-7,0)$, it is given by $\boxed{\begin{aligned} x &= t \\ y &= -7+2t \\ z &= 2t \end{aligned}}$



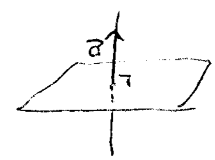


(23) Let $P=(1,1,-1)$, $Q=(2,0,2)$, $R=(0,-2,1)$ and
$\vec{a} = \overrightarrow{PQ} = \langle 1,-1,3\rangle$ and $\vec{b} = \overrightarrow{PR} = \langle -1,-3,2\rangle$.

Then $\vec{n} = \vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 3 \\ -1 & -3 & 2 \end{vmatrix} = 7\vec{i} - 5\vec{j} + (-4)\vec{k} = \langle 7,-5,-4\rangle$ is a normal vector,

so $7x-5y-4z = 7(1)-5(1)-4(-1)$ gives $\boxed{7x-5y-4z=6}$

(25) The plane is perpendicular to the line $x=5+t$, $y=1+3t$, $z=4t$; so the direction vector $\vec{a} = \langle 1,3,4\rangle$ for the line is a normal vector for the plane: $x+3y+4z = 2+3(4)+4(5)$ gives $\boxed{x+3y+4z=34}$

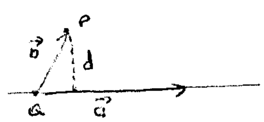


(29) L_1: $x=-1+t$, $y=2+t$, $z=1-t$ has direction vector $\vec{a}_1 = \langle 1,1,-1\rangle$
L_2: $x=1-4s$, $y=1+2s$, $z=2-2s$ has direction vector $\vec{a}_2 = \langle -4,2,-2\rangle = -2\langle 2,-1,1\rangle$
so a normal vector $\vec{n}$ for the plane is given by

$$\vec{n} = \vec{a}_1 \times \left(-\tfrac{1}{2}\vec{a}_2\right) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 2 & -1 & 1 \end{vmatrix} = 0\vec{i} - 3\vec{j} + (-3)\vec{k} = \langle 0,-3,-3\rangle = -3\langle 0,1,1\rangle$$

Since $P=(-1,2,1)$ is on L_1, the plane has equation $y+z=2+1$ so $\boxed{y+z=3}$

(34) Find the distance from $P(0,0,0)$ to the line $x=5+3t$, $y=5+4t$, $z=-3-5t$



Let $Q=(5,5,-3)$, $\vec{a} = \langle 3,4,-5\rangle$, $\vec{b} = \overrightarrow{QP} = \langle -5,-5,3\rangle$.

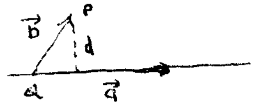
Then $d^2 = |\vec{b}|^2 - (\mathrm{comp}_{\vec{a}}\vec{b})^2 = \vec{b}\cdot\vec{b} - \left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)^2 = 59 - \left(\dfrac{50}{\sqrt{50}}\right)^2 = 59-50 = 9$,

so $d = \boxed{3}$

OR $d = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}|}$ where $\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 4 & -5 \\ -5 & -5 & 3 \end{vmatrix} = -13\vec{i} - (-16)\vec{j} + 5\vec{k} = \langle -13,16,5\rangle$,

so $d = \dfrac{\sqrt{13^2+16^2+5^2}}{\sqrt{3^2+4^2+5^2}} = \dfrac{\sqrt{169+256+25}}{\sqrt{9+16+25}} = \dfrac{\sqrt{450}}{\sqrt{50}} = \sqrt{9} = \boxed{3}$

(37) Find the distance from $P(3,-1,4)$ to the line $x=4-t$, $y=3+2t$, $z=-5+3t$



Let $Q=(4,3,-5)$, $\vec{a} = \langle -1,2,3\rangle$, $\vec{b} = \overrightarrow{QP} = \langle -1,-4,9\rangle$

Then $d^2 = |\vec{b}|^2 - (\mathrm{comp}_{\vec{a}}\vec{b})^2 = \vec{b}\cdot\vec{b} - \left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)^2 = 98 - \left(\dfrac{20}{\sqrt{14}}\right)^2 = 98 - \dfrac{400}{14} = \dfrac{486}{7}$,

so $d = \boxed{\sqrt{\dfrac{486}{7}}} = \boxed{\dfrac{9\sqrt{6}}{\sqrt{7}}} = \boxed{\dfrac{9\sqrt{42}}{7}}$

OR $d = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}|}$ where $\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 3 \\ -1 & -4 & 9 \end{vmatrix} = 30\vec{i} - (-6)\vec{j} + 6\vec{k} = \langle 30,6,6\rangle = 6\langle 5,1,1\rangle$,

so $d = \dfrac{6\sqrt{25+1+1}}{\sqrt{1+4+9}} = \boxed{\dfrac{6\sqrt{27}}{\sqrt{14}}} = \boxed{\dfrac{18\sqrt{3}}{\sqrt{14}}} = \dfrac{18\sqrt{3}\sqrt{14}}{14} = \boxed{\dfrac{9}{7}\sqrt{42}}$

MATH 21C SOLUTIONS - SEC. 12.5

(45) The planes $x+2y+6z=1$ and $x+2y+6z=10$ are parallel, so we can take the distance from a point on one plane to the other plane: the distance from $P(1,0,0)$ to the plane $x+2y+6z=10$ is given by

$$D = \frac{|1+2(0)+6(0)-10|}{\sqrt{1^2+2^2+6^2}} = \frac{|-9|}{\sqrt{41}} = \boxed{\frac{9}{\sqrt{41}}} = \boxed{\frac{9}{41}\sqrt{41}}$$

(47) $x+y=1$, $2x+y-2z=2$ have normal vectors $\vec{n_1}=\langle 1,1,0\rangle$ and $\vec{n_2}=\langle 2,1,-2\rangle$,

so $\cos\theta = \frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|} = \frac{3}{\sqrt{2}\cdot\sqrt{9}} = \frac{1}{\sqrt{2}}$. Then $\theta=\frac{\pi}{4}$, so $\alpha=\theta=\boxed{\frac{\pi}{4}}$ (since $0\le\theta\le\frac{\pi}{2}$)

(53) $x=1-t$, $y=3t$, $z=1+t$ intersects the plane $2x-y+3z=6$ where

$2(1-t)-3t+3(1+t)=6$, so $2-2t-3t+3+3t=6$, $-2t=1$, $t=-1/2$

Then $x=3/2$, $y=-3/2$, $y=1/2$: $\boxed{(3/2,-3/2,1/2)}$

(59) $x-2y+4z=2$, $x+y-2z=5$ have normal vectors $\vec{n_1}=\langle 1,-2,4\rangle$ and $\vec{n_2}=\langle 1,1,-2\rangle$

Then $\vec{a}=\vec{n_1}\times\vec{n_2}$ is a direction vector for the line of intersection,

where $\vec{n_1}\times\vec{n_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 4 \\ 1 & 1 & -2 \end{vmatrix} = \langle 0,6,3\rangle = 3\langle 0,2,1\rangle$.

To find a point on the line, let $z=0$ to get the equations $x-2y=2$ and $x+y=5$. Subtracting gives $-3y=-3$, so $y=1$ and then $x=4$;

so $(4,1,0)$ is on the line.

Then $\boxed{x=4,\ y=1+2t,\ z=t}$ is the line of intersection.

OR $x-2y+4z=2$
$x+y-2z=5$

1) Adding Eq. 1 to 2(Eq. 2) gives
$$\begin{array}{r} x-2y+4z=2 \\ 2x+2y-4z=10 \\ \hline 3x \quad\quad =12 \end{array}$$
so $x=4$

2) Substituting into Eq. 2 gives $4+y-2z=5$, so $y=2z+1$

3) Let $z=t$ to get $\boxed{x=4,\ y=2t+1,\ z=t}$

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(61) L_1: $x=3+2t$, $y=-1+4t$, $z=2-t$
L_2: $x=1+4s$, $y=1+2s$, $z=-3+4s$
L_3: $x=3+2r$, $y=2+r$, $z=-2+2r$

have direction vectors $\vec{a_1}=\langle 2,4,-1\rangle$, $\vec{a_2}=\langle 4,2,4\rangle$, $\vec{a_3}=\langle 2,1,2\rangle$

A) Only $\vec{a_2}$ and $\vec{a_3}$ are parallel, so L_2 and L_3 are parallel.*

B) To see if L_1 and L_2 intersect, we can solve
$3+2t=1+4s$, $-1+4t=1+2s$, $2-t=-3+4s$ to get
$t-2s=-1$, $2t-s=1$, $-t-4s=-5$.

Solving the first pair of equations gives
$$\begin{array}{r} 2t-4s=-2 \\ -\ \ 2t-s=1 \\ \hline -3s=-3 \end{array}$$
$s=1$, $t=1$

Since this satisfies the 3rd equation, L_1 and L_2 intersect at $(5,3,1)$

C) To see if L_1 and L_3 intersect, we can solve
$3+2t=3+2r$, $-1+4t=2+r$, $2-t=-2+2r$ to get
$t=r$, $4t-r=3$, $-t-2r=-4$

Solving the first pair of equations gives $3r=3$ so $t=r=1$,
but this does not satisfy the 3rd equation.

Therefore L_1 and L_3 do not intersect (and are not parallel),
so L_1 and L_3 are skew lines.

* (Notice that $L_2\ne L_3$, since $(1,1,-3)$ is on L_2 but is not on L_3.)