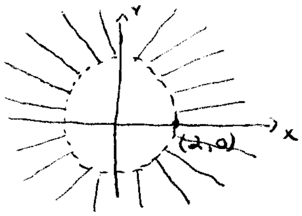
MATH 21C SOLUTIONS - SEC. 14.1 AND 14.2

14.1 - 6 $f(x,y) = \ln(x^2+y^2-4)$ is defined where $x^2+y^2-4>0$ or $x^2+y^2>4$:

a) Boundary: $x^2+y^2=4$

b) Test point $(0,0)$: $0^2+0^2<4$, so $(0,0)$ is not in the domain.

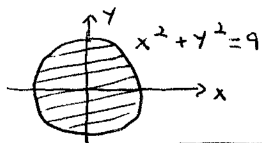


Domain: $\boxed{\text{All points outside the circle } x^2+y^2=4}$

24 $f(x,y) = \sqrt{9-x^2-y^2}$

a) f is defined where $9-x^2-y^2 \geq 0$ or $x^2+y^2 \leq 9$:

Boundary: $x^2+y^2=9$ Test point $(0,0)$: $0^2+0^2 \leq 9$, so $(0,0)$ is in the domain.



Domain: $\boxed{\text{All points inside or on the circle } x^2+y^2=9}$

b) Range: $\boxed{[0,3]}$

c) $f(x,y)=c$ gives $\sqrt{9-x^2-y^2}=c$ or $x^2+y^2=9-c^2$, so the level curves are circles for $0 \leq c < 3$, and the origin for $c=3$.

d) The circle $x^2+y^2=9$ e) Closed (since it contains its boundary)

f) The domain is bounded

14.2 - 14 $\displaystyle \lim_{(x,y)\to(1,1)} \frac{x^2-y^2}{x-y} = \lim_{(x,y)\to(1,1)} \frac{\cancel{(x-y)}(x+y)}{\cancel{x-y}} = \lim_{(x,y)\to(1,1)} (x+y) = \boxed{2}$

20 $\displaystyle \lim_{(x,y)\to(4,3)} \frac{\sqrt{x}-\sqrt{y+1}}{x-y-1} \cdot \frac{\sqrt{x}+\sqrt{y+1}}{\sqrt{x}+\sqrt{y+1}} = \lim_{(x,y)\to(4,3)} \frac{\cancel{x-y-1}}{\cancel{(x-y-1)}(\sqrt{x}+\sqrt{y+1})}$

$\displaystyle = \lim_{(x,y)\to(4,3)} \frac{1}{\sqrt{x}+\sqrt{y+1}} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$

42 $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2}$

1) On the x-axis, $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2} = \lim_{x\to 0} \frac{x^4}{x^4} = \lim_{x\to 0} 1 = \underline{1}$

2) On the y-axis, $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2} = \lim_{y\to 0} \frac{0}{y^2} = \lim_{y\to 0} 0 = \underline{0}$

Therefore $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2}$ $\boxed{\text{does not exist}}$.

48 $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$

1) On the x-axis, $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2} = \lim_{x\to 0} \frac{0}{x^4} = \lim_{x\to 0} 0 = \underline{0}$

2) On the parabola $y=x^2$, $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2} = \lim_{x\to 0} \frac{x^4}{2x^4} = \lim_{x\to 0} \frac{1}{2} = \underline{\frac{1}{2}}$

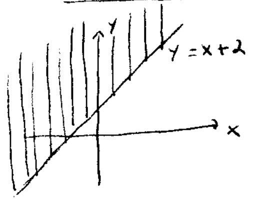
Therefore $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$ $\boxed{\text{does not exist}}$.

MATH 21C SOLUTIONS - SEC. 14.1 AND 14.2

14.1 - (5) $f(x,y) = \sqrt{y - x - 2}$

f is defined where $y - x - 2 \geq 0$ or $y \geq x + 2$:

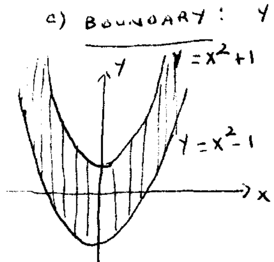
a) Boundary: $y = x + 2$ b) Test $(0,0)$: $0 < 0 + 2$, so $(0,0)$ is NOT in the domain



Domain: $\boxed{\text{ALL POINTS ON OR ABOVE THE LINE } y = x + 2}$

(9) $f(x,y) = \cos^{-1}(y - x^2)$ is defined where $-1 \leq y - x^2 \leq 1$ or $x^2 - 1 \leq y \leq x^2 + 1$

a) Boundary: $y = x^2 - 1$ and $y = x^2 + 1$ b) Test $(0,0)$: $-1 \leq 0 \leq 1$, so $(0,0)$ is in the domain



Domain: $\boxed{\text{ALL POINTS ON OR ABOVE THE PARABOLA } y = x^2 - 1 \text{ AND ON OR BELOW THE PARABOLA } y = x^2 + 1}$

(23) $f(x,y) = \dfrac{1}{\sqrt{16 - x^2 - y^2}}$

(a) f is defined where $16 - x^2 - y^2 > 0$ or $x^2 + y^2 < 16$

1) Boundary: $x^2 + y^2 = 16$ 2) Test $(0,0)$: $0 + 0 < 16$, so $(0,0)$ is in the domain

Domain: $\boxed{\text{ALL POINTS INSIDE THE CIRCLE } x^2 + y^2 = 16}$

14.2 - (21) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} = \lim_{u\to 0} \frac{\sin u}{u} = \boxed{1}$

(Let $u = x^2 + y^2$)

(23) $\displaystyle \lim_{(x,y)\to(1,-1)} \frac{x^3+y^3}{x+y} = \lim_{(x,y)\to(1,-1)} \frac{\cancel{(x+y)}(x^2-xy+y^2)}{\cancel{x+y}} = \lim_{(x,y)\to(1,-1)} (x^2 - xy + y^2) = \boxed{3}$

(45) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x-y}{x+y}$

a) On the x-axis,

$\displaystyle \lim_{(x,y)\to(0,0)} \frac{x-y}{x+y} = \lim_{x\to 0} \frac{x}{x} = \lim_{x\to 0} 1 = \underline{1}$

b) On the y-axis,

$\displaystyle \lim_{(x,y)\to(0,0)} \frac{x-y}{x+y} = \lim_{y\to 0} \frac{-y}{y} = \lim_{y\to 0} (-1) = \underline{-1}$

Therefore $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x-y}{x+y}$ $\boxed{\text{DOES NOT EXIST}}$.