(11) $f(x,y) = 2x^2 + 3xy + 4y^2 - 5x + 2y$

A) $f_x = 4x + 3y - 5 = 0 \xrightarrow{\times 3} 12x + 9y = 15$

$f_y = 3x + 8y + 2 = 0 \xrightarrow{\times 4} \ominus\ 12x + 32y = -8$

$-23y = 23$ $\underline{y = -1}$ $4x + 3(-1) = +5$ $4x = 8$ $\underline{x = 2}$

so $\underline{(2,-1)}$ is the only critical point

B) $f_{xx} = 4$ $f_{xy} = 3$ $f_{yy} = 8$ $D = f_{xx}f_{yy} - (f_{xy})^2 = 32 - 9 = 23$

since $D > 0$ and $f_{xx} > 0$, f has a $\underline{\text{LOCAL MIN.}}$ at $(2,-1)$.

(13) $f(x,y) = x^3 - y^3 - 2xy + 6$

A) $f_x = 3x^2 - 2y = 0$ $2y = 3x^2$ so $\underline{y = \frac{3}{2}x^2}$

$f_y = -3y^2 - 2x = 0$ $2x = -3y^2$

$2x = -3\left(\frac{3}{2}x^2\right)^2 = -3\left(\frac{9}{4}x^4\right)$, so $8x = -27x^4$, $27x^4 + 8x = 0$, $x(27x^3 + 8) = 0$,

$\underline{x = 0}$ or $x^3 = \frac{-8}{27}$, $\underline{x = \frac{2}{-3}}$ since $y = \frac{3}{2}x^2$,

$\underline{(0,0)}$ and $\underline{\left(-\frac{2}{3}, \frac{2}{3}\right)}$ are the critical points

B) $f_{xx} = 6x$ $f_{xy} = -2$ $f_{yy} = -6y$

	f_{xx}	f_{xy}	f_{yy}	D
$(0,0)$	0	-2	0	-4
$\left(-\frac{2}{3}, \frac{2}{3}\right)$	-4	-2	-4	12

$\underline{\text{SADDLE PT. AT } (0,0)}$

$\underline{\text{LOCAL MAX. AT } \left(-\frac{2}{3}, \frac{2}{3}\right)}$

(15) $f(x,y) = 6x^2 - 2x^3 + 3y^2 + 6xy$

A) $f_x = 12x - 6x^2 + 6y = 0$

$f_y = 6y + 6x = 0$ so $\underline{y = -x}$

$12x - 6x^2 - 6x = 0$ so $6x - 6x^2 = 0$, $6x(1-x) = 0$, $\underline{x = 0}$ or $\underline{x = 1}$

$\underline{\text{CRITICAL POINTS:}}$ $\underline{(0,0) \text{ and } (1,-1)}$

B) $f_{xx} = 12 - 12x$ $f_{xy} = 6$ $f_{yy} = 6$

	f_{xx}	f_{xy}	f_{yy}	D
$(0,0)$	12	6	6	36
$(1,-1)$	0	6	6	-36

$\underline{\text{LOCAL MIN. AT } (0,0)}$

$\underline{\text{SADDLE PT. AT } (1,-1)}$

(17) $f(x,y) = x^3 + 3xy^2 - 15x + y^3 - 15y$

A) $f_x = 3x^2 + 3y^2 - 15 = 0$

$f_y = 6xy + 3y^2 - 15 = 0$

$3x^2 - 6xy = 0 \quad 3x(x-2y) = 0$ so $\underline{x=0}$ or $\underline{x=2y}$

1) IF $\underline{x=0}$, $3y^2 = 15$ so $y^2 = 5$, $\underline{y = \pm\sqrt{5}}$

2) IF $\underline{x=2y}$, $3(2y)^2 + 3y^2 - 15 = 0$, $15y^2 = 15$, $y^2 = 1$, $\underline{y = \pm 1}$

CRITICAL POINTS: $(0,\sqrt{5}), (0,-\sqrt{5}), (2,1), (-2,-1)$

3) $f_{xx} = 6x \quad f_{xy} = 6y \quad f_{yy} = 6x + 6y$

	f_{xx}	f_{xy}	f_{yy}	D
$(0,\sqrt{5})$	0	$6\sqrt{5}$	$6\sqrt{5}$	-180
$(0,-\sqrt{5})$	0	$-6\sqrt{5}$	$-6\sqrt{5}$	-180
$(2,1)$	12	6	18	180
$(-2,-1)$	-12	-6	-18	180

SADDLE PTS AT $(0,\sqrt{5})$ AND $(0,-\sqrt{5})$

LOCAL MIN. AT $(2,1)$

LOCAL MAX. AT $(-2,-1)$

(19) $f(x,y) = 4xy - x^4 - y^4$

A) $f_x = 4y - 4x^3 = 0 \qquad y = x^3$

$f_y = 4x - 4y^3 = 0 \qquad x = y^3$

$x = (x^3)^3 = x^9$, $x^9 - x = 0$, $x(x^8 - 1) = 0$, $\underline{x=0}$ or $x^8 = 1$, $\underline{x = \pm 1}$

Since $y = x^3$, THE CRITICAL POINTS ARE $(0,0), (1,1), (-1,-1)$.

B) $f_{xx} = -12x^2 \quad f_{xy} = 4 \quad f_{yy} = -12y^2$

	f_{xx}	f_{xy}	f_{yy}	D
$(0,0)$	0	4	0	-16
$(1,1)$	-12	4	-12	128
$(-1,-1)$	-12	4	-12	128

SADDLE PT. AT $(0,0)$

LOCAL MAX. AT $(1,1)$

LOCAL MAX. AT $(-1,-1)$

(16) $f(x,y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$

A) $f_x = 3x^2 + 6x = 0$ $\quad 3x(x+2) = 0$ $\quad \underline{x=0}$ OR $\underline{x=-2}$

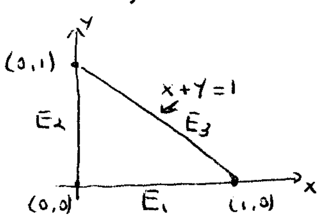
$f_y = 3y^2 - 6y = 0$ $\quad 3y(y-2) = 0$ $\quad \underline{y=0}$ OR $\underline{y=2}$

<u>CRITICAL POINTS: (0,0), (0,2), (-2,0), (-2,2)</u>

B) $f_{xx} = 6x + 6$ $\quad f_{xy} = 0$ $\quad f_{yy} = 6y - 6$

	f_{xx}	f_{xy}	f_{yy}	D	
(0,0)	6	0	-6	-36	<u>SADDLE PT.</u> AT (0,0)
(0,2)	6	0	6	36	<u>LOCAL MIN.</u> AT (0,2)
(-2,0)	-6	0	-6	36	<u>LOCAL MAX.</u> AT (-2,0)
(-2,2)	-6	0	6	-36	<u>SADDLE PT.</u> AT (-2,2)

(38) $f(x,y) = 4x - 8xy + 2y + 1$



a) $f_x = 4 - 8y = 0$ $\quad \underline{y = \frac{1}{2}}$

$f_y = -8x + 2 = 0$ $\quad \underline{x = \frac{1}{4}}$

SO $\underline{\left(\frac{1}{4}, \frac{1}{2}\right)}$ IS THE ONLY CRITICAL POINT IN THE INTERIOR.

b) ON THE BOUNDARY, AN EXTREMUM COULD OCCUR AT THE ENDPOINTS <u>(0,0), (1,0), AND (0,1)</u>;

AND 1) <u>ON E_1</u>, $y=0$: IF $g(x) = f(x,0) = 4x+1$, $g'(x) = 4 \neq 0$; SO THERE ARE NO CRITICAL POINTS ON E_1.

2) <u>ON E_2</u>, $x=0$: IF $h(y) = f(0,y) = 2y+1$, $h'(y) = 2 \neq 0$; SO THERE ARE NO CRITICAL POINTS ON E_2.

3) <u>ON E_3</u>, $y = 1-x$: IF $g(x) = f(x, 1-x) = 4x - 8x(1-x) + 2(1-x) + 1$
$= 8x^2 - 6x + 3$,

THEN $g'(x) = 16x - 6 = 0$ IF $\underline{x = 3/8}$, $\underline{y = 5/8} \leftarrow (1 - 3/8)$

THEREFORE $\underline{\left(\frac{3}{8}, \frac{5}{8}\right)}$ IS THE ONLY CRITICAL POINT ON E_3.

c) $\boxed{f(0,0) = 1 \text{ IS THE MIN.}}$

$f(0,1) = 3$

$\boxed{f(1,0) = 5 \text{ IS THE MAX.}}$

$f\left(\frac{1}{4}, \frac{1}{2}\right) = 2$

$f\left(\frac{3}{8}, \frac{5}{8}\right) = \frac{15}{8}$

(44) a) $f(x,y) = x^2y^2$ $\quad f(0,0) = 0$ IS A $\boxed{\text{LOCAL MIN.}}$, SINCE $f(x,y) = x^2y^2 \geq 0$ FOR ALL (x,y).

b) $f(x,y) = 1 - x^2y^2$

$f(0,0) = 1$ IS A $\boxed{\text{LOCAL MAX.}}$, SINCE $f(x,y) = 1 - (xy)^2 \leq 1$ FOR ALL (x,y).

c) $f(x,y) = xy^2$

<u>ON THE LINE $y = x$</u>,

1) $f(x,x) = x^3 > 0$ FOR $x > 0$, SO $f(0,0) = 0$ IS <u>NOT</u> A LOCAL <u>MAX.</u>

AND 2) $f(x,x) = x^3 < 0$ FOR $x < 0$, SO $f(0,0) = 0$ IS <u>NOT</u> A LOCAL <u>MIN.</u>

THEREFORE (0,0) IS A $\boxed{\text{SADDLE POINT}}$.