MATH 21C SOLUTIONS – SEC. 12.1 AND 12.2 AND 12.3

12.1 – 56 $x^2+y^2+z^2-6y+8z=0$

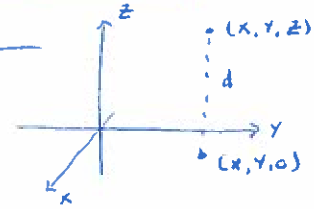
$x^2+(y^2-6y+9)+(z^2+8z+16)=0+9+16$

$x^2+(y-3)^2+(z+4)^2=25$ $\boxed{C\ (0,3,-4)}$ $\boxed{r=5}$

60 a) DISTANCE TO XY-PLANE: $|z|$ ←

b) DISTANCE TO YZ-PLANE: $|x|$

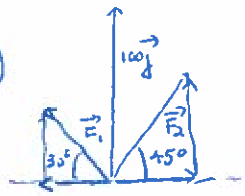
c) DISTANCE TO XZ-PLANE: $|y|$



$d=\sqrt{(x-x)^2+(y-y)^2+(z-0)^2}$

$=\sqrt{z^2}=|z|$

12.2 – 45



$\vec{F}_1+\vec{F}_2=100\vec{j}$ IF $M_1=|\vec{F}_1|$ AND $M_2=|\vec{F}_2|$,

$\vec{F}_1=\langle -M_1\cos 30°, M_1\sin 30°\rangle=\left\langle -\frac{M_1\sqrt{3}}{2}, \frac{M_1}{2}\right\rangle$ AND

$\vec{F}_2=\langle M_2\cos 45°, M_2\sin 45°\rangle=\left\langle \frac{M_2\sqrt{2}}{2}, \frac{M_2\sqrt{2}}{2}\right\rangle$

SINCE $\vec{F}_1+\vec{F}_2=100\vec{j}$, $\left\langle -\frac{M_1\sqrt{3}}{2}+\frac{M_2\sqrt{2}}{2}, \frac{M_1}{2}+\frac{M_2\sqrt{2}}{2}\right\rangle=\langle 0,100\rangle$.

THEN $-\frac{M_1\sqrt{3}}{2}+\frac{M_2\sqrt{2}}{2}=0$, SO $M_2\sqrt{2}=M_1\sqrt{3}$ AND $M_2=\frac{M_1\sqrt{3}}{\sqrt{2}}=\frac{M_1\sqrt{6}}{2}$

SINCE $\frac{M_1}{2}+\frac{M_2\sqrt{2}}{2}=100$, $M_1+M_2\sqrt{2}=200$ SO $M_1+M_1\sqrt{3}=200$,

$M_1(1+\sqrt{3})=200$, $\boxed{M_1=\frac{200}{1+\sqrt{3}}}$ N AND $\boxed{M_2=\frac{100\sqrt{6}}{1+\sqrt{3}}}$ N.

THEREFORE $\boxed{\vec{F}_1=\left\langle -\frac{100\sqrt{3}}{1+\sqrt{3}}, \frac{100}{1+\sqrt{3}}\right\rangle}$ AND $\boxed{\vec{F}_2=\left\langle \frac{100\sqrt{3}}{1+\sqrt{3}}, \frac{100\sqrt{3}}{1+\sqrt{3}}\right\rangle}$

12.3 – 4 $\vec{v}=2\vec{i}+10\vec{j}-11\vec{k}=\langle 2,10,-11\rangle$

$\vec{u}=2\vec{i}+2\vec{j}+\vec{k}=\langle 2,2,1\rangle$

a) $\vec{v}\cdot\vec{u}=4+20-11=13$, $|\vec{v}|=\sqrt{4+100+121}=15$, $|\vec{u}|=\sqrt{4+4+1}=3$

b) $\cos\theta=\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}=\frac{13}{15\cdot 3}=\frac{13}{45}$

c) $\operatorname{comp}_{\vec{v}}\vec{u}=\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}=\frac{13}{15}$

d) $\operatorname{proj}_{\vec{v}}\vec{u}=\left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v}=\frac{13}{225}\vec{v}=\left\langle \frac{26}{225}, \frac{130}{225}, -\frac{143}{225}\right\rangle$

(CHECK: $\vec{u}-\operatorname{proj}_{\vec{v}}\vec{u}=\frac{1}{225}\langle 424,320,368\rangle$,

AND $\langle 424,320,368\rangle\cdot\vec{v}=\langle 424,320,368\rangle\cdot\langle 2,10,-11\rangle=0$)

MATH 21C

SOLUTIONS – SEC. 12.1 AND 12.3

12.1 – 59 a) THE CLOSEST POINT ON THE x-AXIS IS $Q(x,0,0)$,

SO $d(P,Q) = \sqrt{(x-x)^2+(y-0)^2+(z-0)^2} = \boxed{\sqrt{y^2+z^2}}$

b) THE CLOSEST POINT ON THE y-AXIS IS $Q(0,y,0)$,

SO $d(P,Q) = \sqrt{(x-0)^2+(y-y)^2+(z-0)^2} = \boxed{\sqrt{x^2+z^2}}$

c) THE CLOSEST POINT ON THE z-AXIS IS $Q(0,0,z)$,

SO $d(P,Q) = \sqrt{(x-0)^2+(y-0)^2+(z-z)^2} = \boxed{\sqrt{x^2+y^2}}$

65 a) THE SPHERE $x^2+(y-3)^2+(z+5)^2=4$ HAS CENTER $C(0,3,-5)$ AND RADIUS 2, SO THE CLOSEST POINT ON THE SPHERE TO THE xy-PLANE IS 2 UNITS ABOVE THE CENTER: $\boxed{(0,3,-3)}$

12.3 – 3 $\vec{v} = \langle 10, 11, -2\rangle$, $\vec{u} = \langle 0,3,4\rangle$

a) $\vec{v}\cdot\vec{u} = 0+33-8 = \boxed{25}$ $\quad |\vec{v}| = \sqrt{10^2+11^2+2^2} = \sqrt{225} = \boxed{15}$ $\quad |\vec{u}| = \sqrt{0^2+3^2+4^2} = \sqrt{25} = \boxed{5}$

b) $\cos\theta = \dfrac{\vec{v}\cdot\vec{u}}{|\vec{v}||\vec{u}|} = \dfrac{25}{15(5)} = \boxed{\dfrac{1}{3}}$

c) $\text{comp}_{\vec{v}}\vec{u} = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{v}|} = \dfrac{25}{15} = \boxed{\dfrac{5}{3}}$ (OR USE $\text{comp}_{\vec{v}}\vec{u} = |\vec{u}|\cos\theta = 5\left(\frac{1}{3}\right) = \boxed{\frac{5}{3}}$)

d) $\text{proj}_{\vec{v}}\vec{u} = \left(\dfrac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v} = \dfrac{25}{225}\vec{v} = \dfrac{1}{9}\vec{v} = \boxed{\left\langle \dfrac{10}{9}, \dfrac{11}{9}, -\dfrac{2}{9}\right\rangle}$

15 a) $\cos\alpha = \dfrac{\vec{v}\cdot\vec{i}}{|\vec{v}||\vec{i}|} = \dfrac{a}{|\vec{v}|\cdot 1} = \dfrac{a}{|\vec{v}|}$, $\quad \cos\beta = \dfrac{\vec{v}\cdot\vec{j}}{|\vec{v}||\vec{j}|} = \dfrac{b}{|\vec{v}|\cdot 1} = \dfrac{b}{|\vec{v}|}$, $\quad \cos\gamma = \dfrac{\vec{v}\cdot\vec{k}}{|\vec{v}||\vec{k}|} = \dfrac{c}{|\vec{v}|\cdot 1} = \dfrac{c}{|\vec{v}|}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \dfrac{a^2}{|\vec{v}|^2} + \dfrac{b^2}{|\vec{v}|^2} + \dfrac{c^2}{|\vec{v}|^2} = \dfrac{a^2+b^2+c^2}{|\vec{v}|^2} = \dfrac{|\vec{v}|^2}{|\vec{v}|^2} = 1$

b) IF $\vec{v} = \langle a,b,c\rangle$ IS A UNIT VECTOR, SO $|\vec{v}|=1$, THEN

$\cos\alpha = a$, $\cos\beta = b$, AND $\cos\gamma = c$ BY PART a).

17 $\vec{v}_1+\vec{v}_2$ AND $\vec{v}_1-\vec{v}_2$ ARE ORTHOGONAL IFF $(\vec{v}_1+\vec{v}_2)\cdot(\vec{v}_1-\vec{v}_2)=0$

IFF $\vec{v}_1\cdot\vec{v}_1 + \cancel{\vec{v}_2\cdot\vec{v}_1} - \cancel{\vec{v}_1\cdot\vec{v}_2} - \vec{v}_2\cdot\vec{v}_2 = 0$ IFF $|\vec{v}_1|^2 = |\vec{v}_2|^2$ IFF $|\vec{v}_1| = |\vec{v}_2|$

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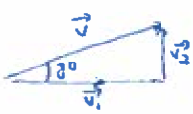

$|\vec{v}_1| = |\vec{v}|\cos 8^\circ = \boxed{1200\cos 8^\circ \text{ FT/SEC}} \approx 1188 \text{ FT/SEC}$

$|\vec{v}_2| = |\vec{v}|\sin 8^\circ = \boxed{1200\sin 8^\circ \text{ FT/SEC}} \approx 167 \text{ FT/SEC}$

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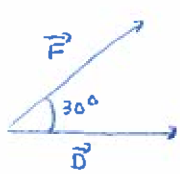

$W = \vec{F}\cdot\vec{D} = |\vec{F}||\vec{D}|\cos\theta = (200)(20)(\cos 30^\circ) = 4000\left(\frac{\sqrt{3}}{2}\right)$

$= \boxed{2000\sqrt{3} \text{ JOULES}} \approx 3464 \text{ J}$