MATH 21C   SOLUTIONS — SEC. 12.4

(7) $\vec{u} = \langle -8,-2,-4\rangle$, $\vec{v} = \langle 2,2,1\rangle$

a) $\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -8 & -2 & -4 \\ 2 & 2 & 1 \end{vmatrix} = \begin{vmatrix} -2 & -4 \\ 2 & 1 \end{vmatrix}\vec{i} - \begin{vmatrix} -8 & -4 \\ 2 & 1 \end{vmatrix}\vec{j} + \begin{vmatrix} -8 & -2 \\ 2 & 2 \end{vmatrix}\vec{k} = \langle 6,0,-12\rangle$

so $|\vec{u}\times\vec{v}| = 6|\langle 1,0,-2\rangle| = \boxed{6\sqrt{5}}$ AND $\dfrac{\vec{u}\times\vec{v}}{|\vec{u}\times\vec{v}|} = \dfrac{\langle 6,0,-12\rangle}{6\sqrt{5}} = \boxed{\left\langle \frac{1}{\sqrt{5}}, 0, \frac{-2}{\sqrt{5}}\right\rangle}$

b) $\vec{v}\times\vec{u} = -\vec{u}\times\vec{v}$,

so $|\vec{v}\times\vec{u}| = |\vec{u}\times\vec{v}| = \boxed{6\sqrt{5}}$ AND $\dfrac{\vec{v}\times\vec{u}}{|\vec{v}\times\vec{u}|} = \dfrac{\langle -6,0,12\rangle}{6\sqrt{5}} = \boxed{\left\langle -\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}}\right\rangle}$

(15) $P(1,-1,2)$, $Q(2,0,-1)$, $R(0,2,1)$

Let $\vec{u} = \vec{PQ} = \langle 1,1,-3\rangle$ AND $\vec{v} = \vec{PR} = \langle -1,3,-1\rangle$,

Then $\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -3 \\ 3 & -1 \end{vmatrix}\vec{i} - \begin{vmatrix} 1 & -3 \\ -1 & -1 \end{vmatrix}\vec{j} + \begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix}\vec{k} = \langle 8,4,4\rangle = 4\langle 2,1,1\rangle$

so a) $A = \frac{1}{2}|\vec{u}\times\vec{v}| = \frac{1}{2}(4)\sqrt{2^2+1^2+1^2} = \boxed{2\sqrt{6}}$

b) IF $\vec{n} = \vec{u}\times\vec{v}$, $\pm\dfrac{\vec{n}}{|\vec{n}|} = \pm\dfrac{4\langle 2,1,1\rangle}{4\sqrt{6}} = \boxed{\pm\left\langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right\rangle}$

(21) $\vec{u} = \langle 2,1,0\rangle$, $\vec{v} = \langle 2,-1,1\rangle$, $\vec{w} = \langle 1,0,2\rangle$

$V = |\vec{u}\cdot(\vec{v}\times\vec{w})| = |(\vec{u}\times\vec{v})\cdot\vec{w}| = \left|\begin{vmatrix} 2 & 1 & 0 \\ 2 & -1 & 1 \\ 1 & 0 & 2 \end{vmatrix}\right| = |2(-2) - 1(3) + 0(1)| = |-7| = \boxed{7}$

OR $\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 0 \\ 2 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix}\vec{i} - \begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix}\vec{j} + \begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix}\vec{k} = \langle 1,-2,-4\rangle$,

so $V = |(\vec{u}\times\vec{v})\cdot\vec{w}| = |1(1) + (-2)(0) + (-4)(2)| = |-7| = \boxed{7}$

(25) $|\vec{\tau}| = |\vec{F}\times\vec{PQ}| = |\vec{F}||\vec{PQ}|\sin\theta = 30(8)\sin 60^\circ = 240\left(\frac{\sqrt{3}}{2}\right) = 120\sqrt{3}$ IN-LB

$= \boxed{10\sqrt{3} \text{ FT-LB}}$

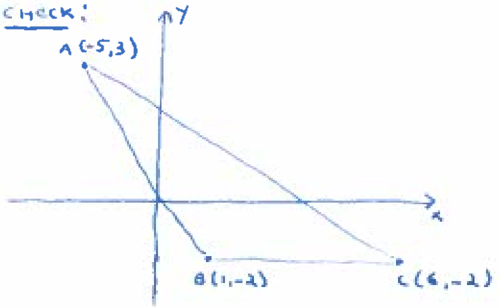
(43) $A(-5,3)$, $B(1,-2)$, $C(6,-2)$

Let $\vec{u} = \vec{BA} = \langle -6,5,0\rangle$ AND $\vec{v} = \vec{BC} = \langle 5,0,0\rangle$,

Then $\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -6 & 5 & 0 \\ 5 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 5 & 0 \\ 0 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} -6 & 0 \\ 5 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} -6 & 5 \\ 5 & 0 \end{vmatrix}\vec{k} = \langle 0,0,-25\rangle$,

so $A = \frac{1}{2}|\vec{u}\times\vec{v}| = \frac{1}{2}\cdot 25 = \boxed{\frac{25}{2}}$

CHECK:



$A = \frac{1}{2}bh = \frac{1}{2}(5)(5) = \frac{25}{2}$

MATH 21C SOLUTIONS – SEC. 12.4 AND 12.5

12.4 – 1. $\vec{u} = \langle 2,-2,-1\rangle$ AND $\vec{v} = \langle 1,0,-1\rangle$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -2 & -1 \\ 1 & 0 & -1 \end{vmatrix} = \begin{vmatrix} -2 & -1 \\ 0 & -1 \end{vmatrix}\vec{i} - \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix}\vec{j} + \begin{vmatrix} 2 & -2 \\ 1 & 0 \end{vmatrix}\vec{k} = \langle 2,1,2\rangle$$

SO $|\vec{u}\times\vec{v}| = \sqrt{2^2+1^2+2^2} = \boxed{3}$ AND $\dfrac{\vec{u}\times\vec{v}}{|\vec{u}\times\vec{v}|} = \boxed{\left\langle \frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right\rangle}$

$\vec{v}\times\vec{u} = -(\vec{u}\times\vec{v})$, SO $|\vec{v}\times\vec{u}| = \boxed{3}$ AND $\dfrac{\vec{v}\times\vec{u}}{|\vec{v}\times\vec{u}|} = \boxed{\left\langle -\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right\rangle}$

40. $A(1,0,-1)$, $B(1,7,2)$, $C(2,4,-1)$, $D(0,3,2)$

LET $\vec{a} = \vec{AC} = \vec{DB} = \langle 1,4,0\rangle$ AND $\vec{b} = \vec{AD} = \vec{CB} = \langle -1,3,3\rangle$.

THEN $A = |\vec{a}\times\vec{b}|$ WHERE $\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 4 & 0 \\ -1 & 3 & 3 \end{vmatrix} = \langle 12,-3,7\rangle$,

SO $A = \sqrt{12^2+3^2+7^2} = \boxed{\sqrt{202}}$.

12.5 – 6. THE LINE PASSES THROUGH $(3,-2,1)$ AND IS PARALLEL TO THE LINE $x = 1+2t$, $y = 2-t$, $z = 3t$:

TAKING $\vec{a} = \langle 2,-1,3\rangle$ GIVES $\boxed{x = 3+2t,\ y = -2-t,\ z = 1+3t}$, t IN $\mathbb{R}$.

22. THE PLANE THROUGH $(1,-1,3)$ PARALLEL TO THE PLANE $3x+y+z=7$:

TAKING $\vec{n} = \langle 3,1,1\rangle$ GIVES $3x+y+z=d$, AND SUBSTITUTING

$x=1$, $y=-1$, $z=3$ GIVES $d=5$; SO $\boxed{3x+y+z=5}$

(OR USE $3(x-1)+1(y-(-1))+1(z-3)=0$ TO GET $\boxed{3x+y+z=5}$)

67. THE LINE $x = 1-2t$, $y = 2+5t$, $z = -3t$ HAS DIRECTION VECTOR $\vec{a} = \langle -2,5,-3\rangle$

AND THE PLANE $2x+y-z=8$ HAS NORMAL VECTOR $\vec{n} = \langle 2,1,-1\rangle$.

SINCE $\vec{a}\cdot\vec{n} = -4+5+3 = 4 \neq 0$, $\vec{a}$ AND $\vec{n}$ ARE NOT ORTHOGONAL

AND THEREFORE THE LINE IS NOT PARALLEL TO THE PLANE.

OR SUBSTITUTING THE PARAMETRIC EQUATIONS FOR THE LINE INTO THE EQUATION OF THE PLANE GIVES

$$2(1-2t) + (2+5t) - (-3t) = 8,$$

SO $2-4t+2+5t+3t = 8$, $4t=4$, $t=1$.

THEREFORE THE LINE AND PLANE INTERSECT AT $(-1,7,-3)$

AND SO THEY ARE NOT PARALLEL.
