(A) APPROXIMATE $\frac{1}{\sqrt[3]{e}}$ USING THE FIRST 5 TERMS OF A MACLAURIN SERIES, AND FIND AN UPPER BOUND FOR THE ERROR IN THE APPROXIMATION

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots, \quad \text{SO}$$

$$\frac{1}{\sqrt[3]{e}} = e^{-1/3} = 1 + \left(-\tfrac{1}{3}\right) + \frac{(-\frac{1}{3})^2}{2!} + \frac{(-\frac{1}{3})^3}{3!} + \frac{(-\frac{1}{3})^4}{4!} + \frac{(-\frac{1}{3})^5}{5!} + \cdots$$

$$= 1 - \frac{1}{3} + \frac{1}{9\cdot 2} - \frac{1}{27\cdot 6} + \frac{1}{81\cdot 24} - \frac{1}{243\cdot 120} + \cdots$$

$$\approx \boxed{1 - \frac{1}{3} + \frac{1}{18} - \frac{1}{162} + \frac{1}{1944}} \quad \text{WITH} \quad |E| < \frac{1}{29{,}160}$$

(B) APPROXIMATE $\int_0^1 e^{-x^2/2}\,dx$ USING THE FIRST 5 NONZERO TERMS OF A MACLAURIN SERIES, AND FIND AN UPPER BOUND FOR THE ERROR IN THE APPROXIMATION.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots, \quad \text{SO}$$

$$e^{-x^2/2} = 1 + \left(-\frac{x^2}{2}\right) + \frac{(-\frac{x^2}{2})^2}{2!} + \frac{(-\frac{x^2}{2})^3}{3!} + \frac{(-\frac{x^2}{2})^4}{4!} + \frac{(-\frac{x^2}{2})^5}{5!} + \cdots$$

$$= 1 - \frac{x^2}{2} + \frac{x^4}{8} - \frac{x^6}{48} + \frac{x^8}{384} - \frac{x^{10}}{3840} + \cdots \quad \text{AND}$$

$$\int_0^1 e^{-x^2/2}\,dx = \int_0^1 \left(1 - \frac{x^2}{2} + \frac{x^4}{8} - \frac{x^6}{48} + \frac{x^8}{384} - \frac{x^{10}}{3840} + \cdots\right) dx$$

$$= \left[x - \frac{x^3}{6} + \frac{x^5}{40} - \frac{x^7}{336} + \frac{x^9}{3456} - \frac{x^{11}}{42{,}240} + \cdots\right]_0^1$$

$$= 1 - \frac{1}{6} + \frac{1}{40} - \frac{1}{336} + \frac{1}{3456} - \frac{1}{42{,}240} + \cdots$$

$$\approx \boxed{1 - \frac{1}{6} + \frac{1}{40} - \frac{1}{336} + \frac{1}{3456}} \quad \text{WITH} \quad |E| < \frac{1}{42{,}240}$$

(C) APPROXIMATE $\int_0^{.8} \frac{x^3}{x^5+2}\,dx$ USING THE FIRST 4 NONZERO TERMS OF A MACLAURIN SERIES, AND FIND AN UPPER BOUND FOR THE ERROR IN THIS ESTIMATE.

$$\frac{x^3}{x^5+2} = \frac{x^3/2}{x^5/2+1} = \frac{x^3/2}{1-(-x^5/2)} = \frac{x^3}{2} - \frac{x^8}{4} + \frac{x^{13}}{8} - \frac{x^{18}}{16} + \frac{x^{23}}{32} - \cdots$$

$$\left(\frac{a}{1-r}, \quad \text{WHERE } a = \frac{x^3}{2},\ r = -\frac{x^5}{2}\right)$$

$$\text{SO} \quad \int_0^{.8} \frac{x^3}{x^5+2}\,dx = \int_0^{.8} \left(\frac{x^3}{2} - \frac{x^8}{4} + \frac{x^{13}}{8} - \frac{x^{18}}{16} + \frac{x^{23}}{32} - \cdots\right) dx$$

$$= \left[\frac{x^4}{8} - \frac{x^9}{36} + \frac{x^{14}}{112} - \frac{x^{19}}{304} + \frac{x^{24}}{768} - \cdots\right]_0^{.8}$$

$$= \frac{(.8)^4}{8} - \frac{(.8)^9}{36} + \frac{(.8)^{14}}{112} - \frac{(.8)^{19}}{304} + \frac{(.8)^{24}}{768} - \cdots$$

$$\approx \boxed{\frac{(.8)^4}{8} - \frac{(.8)^9}{36} + \frac{(.8)^{14}}{112} - \frac{(.8)^{19}}{304}} \quad \text{WITH} \quad |E| < \frac{(.8)^{24}}{768}$$

(D) APPROXIMATE $\sqrt{104}$ USING THE FIRST 4 TERMS OF A TAYLOR SERIES, AND FIND AN UPPER BOUND FOR THE ERROR IN THIS ESTIMATE.

LET $f(x)=\sqrt{x}$ AND $a=100$.

$f(x) = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f''(x) = -\frac{1}{4}x^{-3/2}$

$f'''(x) = \frac{3}{8}x^{-5/2}$

$f^{(4)}(x) = -\frac{15}{16}x^{-7/2}$

n	$f^{(n)}(a)$	$\frac{f^{(n)}(a)}{n!}$
0	10	10
1	$1/20$	$1/20$
2	$-1/4{,}000$	$-1/8{,}000$
3	$3/800{,}000$	$\frac{1}{1{,}600{,}000}$
4	$-15/160{,}000$	$-\frac{5}{1{,}280{,}000{,}000}$

$$\sqrt{x} = 10 + \frac{1}{20}(x-100) - \frac{1}{8{,}000}(x-100)^2 + \frac{1}{1{,}600{,}000}(x-100)^3 - \frac{1}{256{,}000{,}000}(x-100)^4 + \dots$$

$$\text{so } \sqrt{104} = 10 + \frac{1}{20}(4) - \frac{1}{8{,}000}(4^2) + \frac{1}{1{,}600{,}000}(4^3) - \frac{1}{256{,}000{,}000}(4^4) + \dots$$

$$\approx \boxed{10 + \frac{1}{5} - \frac{1}{500} + \frac{1}{25{,}000}} = \boxed{10.198040} \quad \text{WITH } |E| < \frac{1}{1{,}000{,}000} = .000001$$

(NOTICE THAT THE SERIES HAS TERMS WHICH ARE ALTERNATING IN SIGN AND DECREASING IN ABSOLUTE VALUE, STARTING WITH THE SECOND TERM.)

(E) APPROXIMATE $\int_0^{.3} \frac{x^2}{\sqrt{1+x^4}}\,dx$ USING THE FIRST 4 NONZERO TERMS OF A MACLAURIN SERIES, AND FIND AN UPPER BOUND FOR THE ERROR IN THIS ESTIMATE.

$$\frac{1}{\sqrt{1+x}} = (1+x)^{-1/2} = 1 + (-\tfrac{1}{2})x + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}x^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}x^3 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})(-\frac{7}{2})}{4!}x^4 + \dots$$

$$= 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \frac{35}{128}x^4 - \dots$$

$$\text{so } \frac{1}{\sqrt{1+x^4}} = 1 - \frac{1}{2}x^4 + \frac{3}{8}x^8 - \frac{5}{16}x^{12} + \frac{35}{128}x^{16} - \dots \quad (\text{SINCE } (x^4)^n = x^{4n})$$

$$\text{THEN } \int_0^{.3} \frac{x^2}{\sqrt{1+x^4}}\,dx = \int_0^{.3} x^2\left(1 - \frac{1}{2}x^4 + \frac{3}{8}x^8 - \frac{5}{16}x^{12} + \frac{35}{128}x^{16} - \dots\right)dx$$

$$= \int_0^{.3}\left(x^2 - \frac{1}{2}x^6 + \frac{3}{8}x^{10} - \frac{5}{16}x^{14} + \frac{35}{128}x^{18} - \dots\right)dx$$

$$= \left[\frac{x^3}{3} - \frac{x^7}{14} + \frac{3}{88}x^{11} - \frac{1}{48}x^{15} + \frac{35}{2432}x^{19} - \dots\right]_0^{.3}$$

$$= \frac{(.3)^3}{3} - \frac{(.3)^7}{14} + \frac{3}{88}(.3)^{11} - \frac{1}{48}(.3)^{15} + \frac{35}{2432}(.3)^{19} - \dots$$

$$\approx \boxed{\frac{(.3)^3}{3} - \frac{(.3)^7}{14} + \frac{3}{88}(.3)^{11} - \frac{(.3)^{15}}{48}} \quad \text{WITH } |E| < \frac{35}{2432}(.3)^{19}$$