MATH 21C DISCUSSION SHEET 3

(1) DETERMINE IF EACH OF THE FOLLOWING SERIES CONVERGES OR DIVERGES, AND JUSTIFY YOUR ANSWERS:

A) $\sum_{n=1}^{\infty} \left(\frac{2n+1}{2n+5}\right)^9$

B) $\sum_{n=1}^{\infty} \left(\frac{2n+1}{2n+5}\right)^{n^2}$

C) $\sum_{n=1}^{\infty} \frac{1}{n^{n+1}}$

D) $\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$

E) $\sum_{n=1}^{\infty} \frac{\ln n}{n + \ln n}$

(2) FINISH THE FOLLOWING PROOF THAT THE HARMONIC SERIES DIVERGES:

PF SUPPOSE INSTEAD THAT $\sum_{n=1}^{\infty} \frac{1}{n}$ CONVERGES, AND LET

$$S = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \cdots \quad \text{THEN}$$

$$S > \frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{6} + \frac{1}{6} + \frac{1}{8} + \frac{1}{8} + \cdots$$

(3) GIVE AN EXAMPLE OF A SERIES $\sum_{n=1}^{\infty} a_n$ SUCH THAT $\sum_{n=1}^{\infty} a_n$ DIVERGES, BUT THE SERIES $(a_1 + a_2) + (a_3 + a_4) + (a_5 + a_6) + \cdots$ CONVERGES.

(4) DETERMINE IF EACH OF THE FOLLOWING SERIES CONVERGES ABSOLUTELY, CONVERGES CONDITIONALLY, OR DIVERGES, AND JUSTIFY YOUR ANSWERS:

A) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^n}{5^n (n+8)!}$

B) $\sum_{n=1}^{\infty} \frac{\cos n}{5^n - 3^n}$

C) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{4^n - 3^{n-1}}{2^n + 3^{n+5}}$

D) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{2^{1/n} + n^{3/n}}$