MATH 21C DISCUSSION SHEET 1

① FIND THE FOLLOWING LIMITS:

A) $\lim_{n\to\infty} \left(1-\frac{1}{2n}\right)^{6n}$

B) $\lim_{n\to\infty} \left(\frac{n+5}{n+2}\right)^{n+8}$ [HINT: FIRST DIVIDE BY n ON THE TOP AND BOTTOM.]

② DEFINE $\{a_n\}$ RECURSIVELY BY $a_1 = 1$ AND $a_n = n a_{n-1}$ FOR $n \geq 2$.

FIND THE FIRST 5 TERMS OF THE SEQUENCE, AND A GENERAL FORMULA FOR a_n.

③ DEFINE $\{a_n\}$ RECURSIVELY BY

$$a_1 = 2, \quad \text{AND} \quad a_n = \frac{1}{2}\left(a_{n-1} + \frac{5}{a_{n-1}}\right) \text{ FOR } n \geq 2.$$

A) FIND THE FIRST 3 TERMS OF THE SEQUENCE,

B) GIVEN THAT $\{a_n\}$ CONVERGES, FIND $\lim_{n\to\infty} a_n$.

④ A SEQUENCE $\{a_n\}$ IS INCREASING (OR NONDECREASING) IF $a_n \leq a_{n+1}$ FOR ALL n, AND $\{a_n\}$ IS BOUNDED ABOVE IF THERE IS A NUMBER M WITH $a_n \leq M$ FOR ALL n.

A) SHOW THAT IF $\{a_n\}$ IS INCREASING AND BOUNDED ABOVE, THEN $\{a_n\}$ CONVERGES. (SEE PP. 579-581)

B) SHOW THAT IF $\{a_n\}$ IS INCREASING AND NOT BOUNDED ABOVE, THEN $\{a_n\}$ DIVERGES TO INFINITY.

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⑤ FOR THE SERIES $\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right)$,

A) USE PROPERTIES OF LOGARITHMS TO REWRITE $\ln\left(1+\frac{1}{n}\right)$.

B) USE PART A) TO FIND A FORMULA FOR S_n.

C) USE PART B) TO FIND THE SUM OF THE SERIES (IF IT CONVERGES), OR TO SHOW THAT IT DIVERGES.

⑥ FOR THE SERIES $\sum_{n=1}^{\infty} \frac{2}{n^2+2n}$,

A) USE PARTIAL FRACTIONS TO REWRITE $\frac{2}{n^2+2n}$.

B) USE PART A) TO FIND A FORMULA FOR S_n.

C) USE PART B) TO FIND THE SUM OF THE SERIES (IF IT CONVERGES), OR TO SHOW THAT IT DIVERGES.

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⑦ IF $\{a_n\}$ IS THE SEQUENCE GIVEN BY

$$1, \quad 1+\frac{1}{3}, \quad 1+\frac{1}{3+\frac{1}{3}}, \quad 1+\frac{1}{3+\frac{1}{3+\frac{1}{3}}}, \quad \ldots$$

A) FIND A RECURSION FORMULA FOR $\{a_n\}$.

B) ASSUMING THAT $\{a_n\}$ CONVERGES, FIND $\lim_{n\to\infty} a_n$.