1. Find the following limits:
   a) \( \lim_{n \to \infty} \left( 1 - \frac{1}{2n} \right)^{6n} \)
   b) \( \lim_{n \to \infty} \left( \frac{n+5}{n+2} \right)^{n+8} \) [HINT: First divide by \( n \) on the top and bottom.]

2. Define \( \{a_n\} \) recursively by \( a_1 = 1 \) and \( a_n = n \cdot a_{n-1} \) for \( n \geq 2 \).
   Find the first 5 terms of the sequence, and a general formula for \( a_n \).

3. Define \( \{a_n\} \) recursively by
   \[ a_1 = 2, \quad a_n = \frac{1}{2} \left( 2a_{n-1} + 5 \right) \text{ for } n \geq 2. \]
   a) Find the first 3 terms of the sequence.
   b) Given that \( \{a_n\} \) converges, find \( \lim_{n \to \infty} a_n \).

4. A sequence \( \{a_n\} \) is **increasing** (or non-decreasing) if \( a_n \leq a_{n+1} \) for all \( n \).
   And \( \{a_n\} \) is **bounded above** if there is a number \( M \) with \( a_n \leq M \) for all \( n \).
   a) Show that if \( \{a_n\} \) is increasing and bounded above,
      then \( \{a_n\} \) converges.
   b) Show that if \( \{a_n\} \) is increasing and not bounded above,
      then \( \{a_n\} \) diverges to infinity.

5. For the series \( \sum_{n=1}^{\infty} \ln \left( 1 + \frac{1}{n} \right) \),
   a) Use properties of logarithms to rewrite \( \ln \left( 1 + \frac{1}{n} \right) \).
   b) Use PART A) to find a formula for \( S_n \).
   c) Use PART B) to find the sum of the series (if it converges),
      or to show that it diverges.

6. For the series \( \sum_{n=1}^{\infty} \frac{2}{n^2 + 2n} \),
   a) Use partial fractions to rewrite \( \frac{2}{n^2 + 2n} \).
   b) Use PART A) to find a formula for \( S_n \).
   c) Use PART B) to find the sum of the series (if it converges),
      or to show that it diverges.

7. If \( \{a_n\} \) is the sequence given by
   \[ 1, 1 + \frac{1}{3}, 1 + \frac{1}{3 + \frac{1}{3}}, 1 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}, \ldots \]
   a) Find a recursion formula for \( \{a_n\} \).
   b) Assuming that \( \{a_n\} \) converges, find \( \lim_{n \to \infty} a_n \).