MATH 21C DISCUSSION SHEET 8

① DETERMINE IF THE FOLLOWING SERIES CONVERGE OR DIVERGE!

A) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$ B) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{2^n}$ C) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$ D) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

② FIND THE INTERVAL OF CONVERGENCE FOR THE POWER SERIES $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{3^n}{(n+1)^2} (x+2)^n$.

③ LET $P = (1,4,6)$, $Q = (-2,5,-1)$, $R = (1,-1,1)$.

a) FIND THE AREA OF TRIANGLE ΔPQR.

b) FIND THE DISTANCE FROM P TO THE LINE QR.

④ FIND THE FOLLOWING LIMITS, OR SHOW THAT THEY DO NOT EXIST:

A) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$ B) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^4}{x^4+3y^4}$

⑤ FIND THE MAXIMUM AND MINIMUM VALUES OF $f(x,y) = x^2 - xy + y^2 + 1$ ON THE TRIANGULAR REGION BOUNDED BY $x=0$, $y=2$, AND $y=2x$.

⑥ IF $f(x,y,z) = \sqrt{x^2+y^2+z^2}$, USE A LINEARIZATION OF f TO APPROXIMATE $\sqrt{(3.1)^2 + (1.9)^2 + (6.2)^2}$.

⑦ IF $\vec{u} = \langle 3,-1,2 \rangle$ AND $\vec{v} = \langle 2,1,-2 \rangle$,

A) FIND THE ANGLE BETWEEN $\vec{u}$ AND $\vec{v}$.

B) FIND THE DIRECTIONAL DERIVATIVE OF $f(x,y,z) = \dfrac{x}{y+z}$ AT $P(4,1,1)$ IN THE DIRECTION OF $\vec{v}$.

⑧ FIND PARAMETRIC EQUATIONS FOR THE LINE OF INTERSECTION OF THE PLANES $3x - 6y - 4z = 15$ AND $6x + y - 2z = 5$.

⑨ ESTIMATE $\displaystyle\int_0^{.5} \sqrt{1+t^2}\,dt$ USING THE FIRST 4 NONZERO TERMS OF A MACLAURIN SERIES, AND FIND AN UPPER BOUND FOR THE ABSOLUTE VALUE OF THE ERROR.

⑩ FIND THE DOMAIN AND RANGE OF $f(x,y) = \dfrac{1}{\sqrt{xy}}$.

⑪ DEFINE $f(x,y) = \begin{cases} \dfrac{xy^2}{x^2+y^4}, & \text{IF } x \neq 0 \\ 0, & \text{IF } x = 0. \end{cases}$

A) SHOW THAT $D_{\vec{u}} f(0,0)$ EXISTS FOR ANY UNIT VECTOR $\vec{u}$.

B) SHOW THAT f IS NOT CONTINUOUS AT $(0,0)$.

⑫ A) PROVE THAT THE SERIES $\displaystyle\sum_{n=0}^{\infty} \frac{\cos n\theta}{2^n}$ CONVERGES FOR EVERY REAL NUMBER θ.

B) FIND THE SUM OF THE SERIES WHEN $\theta = \pi$ AND WHEN $\theta = \frac{\pi}{2}$.

C) FIND THE SUM OF THE SERIES FOR ANY REAL NUMBER θ.