MATH 21C DISCUSSION SHEET 7

① IF $f(x,y) = \dfrac{1}{\ln(y^2 - x)}$, A) DESCRIBE AND SKETCH THE DOMAIN OF f.
B) FIND THE RANGE OF f.

② FIND $\lim_{(x,y)\to(0,0)} \dfrac{xy^2}{x^2+y^4}$, OR SHOW THAT THIS LIMIT DOES NOT EXIST.

③ IF $g(t) = f(t^2+3t, t^3-t)$, WHERE f HAS CONTINUOUS FIRST PARTIALS, FIND $g'(2)$ IN TERMS OF THE PARTIAL DERIVATIVES OF f.

④ A) FIND AN EQUATION OF THE TANGENT PLANE TO THE SURFACE $z = 2x^2 + 3y^2$ AT $P(2,-1,11)$, AND SIMPLIFY YOUR ANSWER.

B) FIND AN EQUATION OF THE TANGENT PLANE TO THE SURFACE WITH EQUATION $x^2y - y^2z + 5z^2 = 19$ AT $P(3,2,1)$.

⑤ LET $f(x,y) = 3x^3 + xy^2 - y^2 - 9x^2$.
A) FIND ALL THE CRITICAL POINTS FOR f.
B) CLASSIFY EACH CRITICAL POINT AS CORRESPONDING TO A LOCAL MAX., LOCAL MIN., OR SADDLE POINT. (CALCULATE THE VALUE OF D AT EACH CRITICAL POINT.)

⑥ LET $f_x(7,9) = -5$ AND $f_y(7,9) = 12$.
A) FIND THE MAXIMAL DIRECTIONAL DERIVATIVE OF f AT $(7,9)$.
B) FIND THE DIRECTIONAL DERIVATIVE OF f AT $(7,9)$ IN THE DIRECTION OF $\vec{v} = \langle 4,3 \rangle$.

⑦ USE LAGRANGE MULTIPLIERS TO FIND THE MAXIMUM VALUE OF $f(x,y,z) = 2x - 4y - 3z$ ON THE ELLIPSOID $2x^2 + (y-3)^2 + 2z^2 = 90$.

⑧ IF $f(x,y) = 6x^2y - 11y^2 - x^4$, DETERMINE IF THE CRITICAL POINT $(0,0)$ CORRESPONDS TO A LOCAL MAX., LOCAL MIN., OR SADDLE POINT (GIVEN THAT $D = 0$ AT $(0,0)$).

⑨ FIND $\lim_{(x,y)\to(0,0)} \dfrac{x^3 + x^2y}{x^2+y^2}$, OR SHOW THAT IT DOES NOT EXIST.

⑩ LET C BE THE CURVE OF INTERSECTION OF THE PLANE $2x + y + 2z = 14$ AND THE CYLINDER $x^2 + y^2 = 25$.
A) FIND PARAMETRIC EQUATIONS FOR THE TANGENT LINE TO C AT $P(3,-4,6)$.
B) FIND THE MAXIMUM VALUE OF $f(x,y,z) = 3x + 4y + 4z$ ON C.