MATH 21C — DISCUSSION SHEET 2

1. A) Does the sequence $\left\{\frac{1}{n}\right\}$ converge or diverge?

   B) Does the series $\sum_{n=1}^{\infty} \frac{1}{n}$ converge or diverge?

2. Find the sum of the series (if it converges), or show that it diverges:

   A) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{7(3^{n+1})}{2^{2n+1}}$

   B) $\sum_{n=0}^{\infty} (-1)^{n} \frac{5^{n}}{6(4^{n+1})}$

3. Let $\sum_{n=1}^{\infty} a_n$ be a series whose sequence of partial sums $\{S_n\}$ is given by $S_n = \frac{5n+3}{n+1}$.

   A) Determine if $\sum_{n=1}^{\infty} a_n$ converges or diverges.

   B) Find a_1, and a formula for a_n for $n \geq 2$.

4. Determine if each of the following series converges or diverges, and justify your answers.

   A) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+2}\right)^2$

   B) $\sum_{n=1}^{\infty} \frac{.8}{n+4}$

   C) $\sum_{n=1}^{\infty} \frac{n^5}{2^n}$

   D) $\sum_{n=1}^{\infty} \frac{n}{(n+1)^2}$

   E) $\sum_{n=1}^{\infty} \left(\frac{n}{n+8}\right)^n$

   F) $\sum_{n=1}^{\infty} \frac{n+1}{n^3+4}$

   G) $\sum_{n=1}^{\infty} \frac{n-6}{n^2}$

   H) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$

   I) $\sum_{n=1}^{\infty} \left(\frac{3n+1}{4n-3}\right)^n$

   J) $\sum_{n=1}^{\infty} \frac{n!\,(n+3)!}{(2n)!}$

5. Find an example of two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ such that

   A) $\sum_{n=1}^{\infty} b_n$ diverges, but $\sum_{n=1}^{\infty} (a_n + b_n)$ converges.

   B) $a_n \leq b_n$ for all n, $\sum_{n=1}^{\infty} b_n$ converges, and $\sum_{n=1}^{\infty} a_n$ diverges.