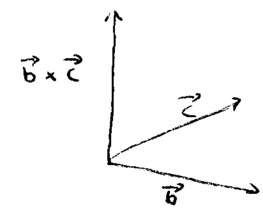
I) The Vector Triple Product



Remark The vector $\vec{a}\times(\vec{b}\times\vec{c})$ is orthogonal to $\vec{b}\times\vec{c}$, so it is in the plane determined by $\vec{b}$ and $\vec{c}$.

Th $\boxed{\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}}$

Ex Let $\vec{a} = \langle 2,3,2\rangle$, $\vec{b} = \langle 4,1,-2\rangle$, $\vec{c} = \langle 3,-5,4\rangle$.

Then $\underline{\vec{a}\times(\vec{b}\times\vec{c})} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

$= -\langle 4,1,-2\rangle - 7\langle 3,-5,4\rangle = \underline{\langle -25, 34, -26\rangle}$.

Remark Notice that $\underline{\vec{a}\times(\vec{b}\times\vec{c}) \neq (\vec{a}\times\vec{b})\times\vec{c}}$ in general since

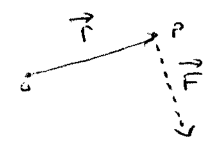
$(\vec{a}\times\vec{b})\times\vec{c} = -\vec{c}\times(\vec{a}\times\vec{b}) = -\left[(\vec{c}\cdot\vec{b})\vec{a} - (\vec{c}\cdot\vec{a})\vec{b}\right] = \underline{(\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}}$

II) Applications of the Cross Product in Physics

① Torque

If a particle P rotates about a point O due to a force $\vec{F}$ applied to P, the resulting torque vector is given by

$\boxed{\vec{\tau} = \vec{r}\times\vec{F}}$ where $\vec{r} = \vec{OP}$



② Angular Momentum

If a particle P rotates about a point O, and its mass is m and its velocity vector is $\vec{v}$, its angular momentum is given by

$\boxed{\vec{L} = \vec{r}\times\vec{p}}$ where $\vec{r} = \vec{OP}$ and $\vec{p} = m\vec{v}$ is the linear momentum,

③ Magnetic Force

Let P be a charged particle moving in a magnetic field, with charge q and velocity vector $\vec{v}$. If $\vec{B}$ is the magnetic field vector at P, then the force exerted by the magnetic field on P is given by

$\boxed{\vec{F} = (q\vec{v})\times\vec{B}}$