10.6. (a) \(1 + \frac{1}{4} - \frac{1}{9} - \frac{1}{16} + \frac{1}{25} + \frac{1}{36} - \frac{1}{49} + \frac{1}{64} + \ldots\) is absolutely convergent.

Since \(\sum_{n=1}^{\infty} \frac{1}{n^2}\) converges because it's a p-series, \(p > 1\).

(b) \(\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^3 + 3}\)

Since \(|R_n| = |S - S_0| < |L_n|\), \(|R_n| < 0.001\) if \(\sum_{n=1}^{\infty} \frac{1}{n^3 + 3}\) converges.

1. \(|L_{n+1}| = \frac{1}{1000}\) \(\iff (n+1)^2 + 3 > 1000 \iff (n+1)^2 > 997 \iff (n+1)^2 > 31.6\)

2. \(|L_{n+1}| \implies n \geq 31\) will guarantee this accuracy.

10.7. (a) \(\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^3 + 3}\)

1. \(L = \lim_{n \to \infty} \frac{|x-1|^n}{n^3 + 3}\)

2. \(L < 1 \iff |x-1| < 1 \iff -1 < x - 1 < 1 \iff 0 < x < 2\)

3. (a) If \(x = 1\), we have \(\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3 + 3}\), which converges by the AST since \(\lim_{n \to \infty} \frac{1}{n^3 + 3} = 0\) and \(\lim_{n \to \infty} \frac{1}{n} = \frac{1}{n^3 + 3}\) for all \(n\).

(b) If \(x = 2\), we have \(\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3 + 3}\), which diverges (p-series, \(p < 1\)).

Thus \([-1, 1]\) is the interval of conv., and \((-1, 1)\) is the radius of conv.

(b) \(\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{\sqrt{n+3}}\)

1. \(L = \lim_{n \to \infty} \frac{|x|^{n+1}}{\sqrt{n+3}}\)

2. \(L < 1 \iff |x| < 1 \iff -1 < x < 1\)

3. (a) If \(x = 1\), we get \(\sum_{n=0}^{\infty} \frac{1}{\sqrt{n+3}}\), which converges (p-series, \(p > 1\)).

(b) If \(x = -1\), we get \(\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+3}}\), which converges since it's absolutely convergent.

Thus \([-1, 1]\) is the interval of conv., and \([-1, 1]\) is the radius of conv.