I) FIND THE FIRST 4 NONZERO TERMS OF THE TAYLOR SERIES FOR EACH OF THE FOLLOWING FUNCTIONS CENTERED AT THE INDICATED VALUES OF a, AND THEN WRITE EACH TAYLOR SERIES IN SUMMATION NOTATION.

① $f(x) = \dfrac{1}{5x-3}$, $a = 1$

② $f(x) = \dfrac{1}{x^2}$, $a = 2$

③ $f(x) = \dfrac{1}{(1-x)^3}$, $a = -1$

④ $f(x) = \dfrac{1}{x}$, $a = 4$

A) USING THE TAYLOR SERIES FORMULA.

B) USING $\dfrac{1}{x} = \dfrac{1}{4+(x-4)} = \dfrac{\frac{1}{4}}{1+\left(\frac{x-4}{4}\right)} = \dfrac{\frac{1}{4}}{1-\left(-\frac{x-4}{4}\right)}$, AND THE FORMULA FOR THE SUM OF A GEOMETRIC SERIES.

⑤ $f(x) = \sqrt{x+3}$, $a = 1$

II) ⑥ FIND THE FIRST 3 NONZERO TERMS OF THE MACLAURIN SERIES FOR $f(x) = \tan x$ BY DIVIDING THE MACLAURIN SERIES FOR $\sin x$ BY THE MACLAURIN SERIES FOR $\cos x$.

⑦ IF YOU WANT TO APPROXIMATE $\ln 3$,

A) EXPLAIN WHY YOU CAN'T USE THE MACLAURIN SERIES FOR $f(x) = \ln(1+x)$ WITH $x = 2$ TO DO THIS.

B) USE THE FIRST 4 NONZERO TERMS OF A MACLAURIN SERIES GIVEN IN THE EXERCISES IN SEC. 10.10 TO DO THIS.

III) ⑧ SHOW THAT ANY CONDITIONALLY CONVERGENT SERIES $\sum_{n=1}^{\infty} a_n$ CAN BE WRITTEN IN THE FORM $\sum_{n=1}^{\infty} (b_n - c_n)$, WHERE $\sum_{n=1}^{\infty} b_n$ AND $\sum_{n=1}^{\infty} c_n$ ARE BOTH DIVERGENT SERIES (WITH NONNEGATIVE TERMS)

<u>HINT</u>: LET $b_n = a_n^+ = \frac{1}{2}(|a_n| + a_n)$ AND $c_n = a_n^- = \frac{1}{2}(|a_n| - a_n)$.