MATH 216 SEC. 10.2 – THEOREMS ON MODIFYING SERIES

TH 1 IF $\sum_{n=1}^{\infty} a_n$ AND $\sum_{n=1}^{\infty} b_n$ ARE CONVERGENT SERIES WITH $\sum_{n=1}^{\infty} a_n = A$ AND $\sum_{n=1}^{\infty} b_n = B$,

THEN 1) $\sum_{n=1}^{\infty} (a_n + b_n) = A + B$ AND 2) $\sum_{n=1}^{\infty} c a_n = cA$ FOR ANY NUMBER c.

REMARK THIS IS TH. 8 IN SEC. 10.2.

TH 2 IF $\sum_{n=1}^{\infty} a_n$ DIVERGES, THEN $\sum_{n=1}^{\infty} c a_n$ DIVERGES FOR ANY $c \neq 0$.

EX $\sum_{n=1}^{\infty} \frac{5}{n}$ DIVERGES, SINCE THE HARMONIC SERIES $\sum_{n=1}^{\infty} \frac{1}{n}$ DIVERGES.

REMARK THIS THEOREM FOLLOWS FROM THE 2ND PART OF TH. 1.

TH 3 IF $\sum_{n=1}^{\infty} a_n$ AND $\sum_{n=1}^{\infty} b_n$ ARE SERIES SUCH THAT ONE CONVERGES AND THE OTHER DIVERGES,

THEN $\sum_{n=1}^{\infty} (a_n \pm b_n)$ DIVERGES.

EX $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{2}{3^n} \right)$ DIVERGES, SINCE

1) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ DIVERGES (p-SERIES WITH $p \leq 1$, SINCE $p = \frac{1}{2}$) AND

2) $\sum_{n=1}^{\infty} \frac{2}{3^n}$ CONVERGES (GEOMETRIC SERIES WITH $-1 < r < 1$, SINCE $r = \frac{1}{3}$)

REMARKS A) THIS THEOREM FOLLOWS FROM THE 1ST PART OF TH. 1.

B) IF $\sum_{n=1}^{\infty} a_n$ AND $\sum_{n=1}^{\infty} b_n$ BOTH DIVERGE, IT DOES NOT NECESSARILY FOLLOW THAT $\sum_{n=1}^{\infty} (a_n \pm b_n)$ DIVERGES. (SEE THE TOP OF P. 590.)

TH 4 IF $\sum_{n=1}^{\infty} a_n$ AND $\sum_{n=1}^{\infty} b_n$ ARE SERIES WITH $a_n = b_n$ FOR $n \geq N$ (FOR SOME N),

THEN EITHER BOTH SERIES CONVERGE OR BOTH SERIES DIVERGE.

COR INSERTING, DELETING, OR CHANGING FINITELY MANY TERMS OF A SERIES DOES NOT AFFECT ITS CONVERGENCE OR DIVERGENCE.

EX A) $\sum_{n=1}^{\infty} \frac{1}{n+4} = \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \cdots$ DIVERGES,

SINCE IT IS THE HARMONIC SERIES WITH 4 TERMS DELETED.

B) $\sum_{n=1}^{\infty} \frac{1}{(n+3)^2} = \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + \cdots$ CONVERGES,

SINCE IT IS A p-SERIES WITH $p > 1$ WITH 3 TERMS DELETED.
