Sec. 14.1 - DEFINITIONS AND EXAMPLES

DEF A CIRCULAR DISC IS THE SET OF ALL POINTS INSIDE A CIRCLE.

DEF LET R BE A REGION IN THE XY-PLANE.

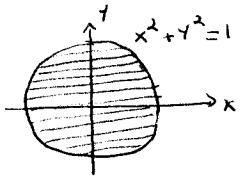
A) 1) A POINT P IN R IS AN INTERIOR POINT OF R IF THERE IS A CIRCULAR DISC CENTERED AT P WHICH IS CONTAINED IN R. THE SET OF ALL INTERIOR POINTS OF R IS CALLED THE INTERIOR OF R.

2) R IS OPEN IF EVERY POINT OF R IS AN INTERIOR POINT.

B) 1) A POINT P IS A BOUNDARY POINT OF R IF EVERY CIRCULAR DISC CENTERED AT P CONTAINS A POINT IN R AND A POINT WHICH IS NOT IN R. THE SET OF ALL BOUNDARY POINTS OF R IS CALLED THE BOUNDARY OF R.

2) R IS CLOSED IF IT CONTAINS ALL ITS BOUNDARY POINTS.

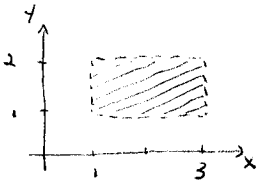
EX A) LET R BE THE CLOSED UNIT DISC $\{(x,y) : x^2+y^2 \le 1\}$,



THE INTERIOR OF R IS THE OPEN UNIT DISC $\{(x,y) : x^2+y^2 < 1\}$, AND THE BOUNDARY OF R IS THE UNIT CIRCLE $\{(x,y) : x^2+y^2 = 1\}$.

R IS CLOSED, SINCE IT CONTAINS ITS BOUNDARY.

B) LET R BE THE RECTANGULAR REGION $\{(x,y) : 1 < x < 3 \text{ AND } 1 < y < 2\}$,



THE INTERIOR OF R IS R ITSELF, SO R IS OPEN.

THE BOUNDARY OF R IS THE RECTANGLE ENCLOSING R (CONSISTING OF 4 LINE SEGMENTS)

REMARK WE CAN EXTEND THESE DEFINITIONS (AND THE ONE BELOW) TO 3 DIMENSIONS IF WE USE THE INSIDE OF A SPHERE IN PLACE OF A CIRCULAR DISC.

DEF A REGION R IN THE XY-PLANE IS BOUNDED IF IT LIES INSIDE SOME CIRCULAR DISC, AND IT IS UNBOUNDED IF IT DOES NOT.

EX A) THE SET OF POINTS ENCLOSED BY A RECTANGLE IS BOUNDED.

B) THE SET OF POINTS INSIDE ANY ELLIPSE IS BOUNDED.

C) THE GRAPH OF $y = x^2$ IS UNBOUNDED.

D) THE SET OF POINTS IN THE FIRST QUADRANT IS UNBOUNDED.

E) THE GRAPH OF $y^2 = x^2(4-x^2)$ IS BOUNDED.