① $A^{-1} = \begin{bmatrix} -7/2 & 5/2 \\ 2 & -1 \end{bmatrix}$

② $A^{-1} = \begin{bmatrix} -2 & -1/2 & 9/2 \\ 3 & 1/2 & -11/2 \\ 1 & 1/2 & -5/2 \end{bmatrix}$

③ $\vec{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ IS NOT IN W, SO W IS <u>NOT</u> A SUBSPACE OF M_{22}.

(<u>OR</u> SHOW W IS NOT CLOSED UNDER ADDITION, <u>OR</u> SHOW W IS NOT CLOSED UNDER SCALAR MULT.)

④ $F(2I) = \det\left(\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right) = 4$, BUT $2F(I) = 2\det(I) = 2 \cdot 1 = 2$; SO $F(2I) \neq 2F(I)$

AND THEREFORE F IS <u>NOT</u> A L.T. (<u>OR</u> SHOW THAT F DOES NOT PRESERVE ADDITION.)

⑤ $[T] = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

⑥ A) $\det(A) \neq 0$ B) RANK $(A) = 6$ C) NULLITY $(A) = 4, 5,$ OR 6

⑦ $\{1, t^2 - 3t, t^3 - 7t\}$

⑧ A) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} \right\}$

RANGE(T) = COL. SPACE (A)

b) $\left\{ \begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$ KER (T) = NULLSPACE (A)

⑨ A) YES, SINCE IT HAS 3 DISTINCT EIGENVALUES.

B) YES, SINCE $\lambda = 0$ IS NOT AN EIGENVALUE FOR A.

⑩ $Ax = 4x$ WITH $x \neq 0 \Rightarrow A^{-1}(Ax) = A^{-1}(4x) \Rightarrow x = 4(A^{-1}x) \Rightarrow A^{-1}x = \frac{1}{4}x$ WITH $x \neq 0$,

SO $\boxed{\frac{1}{4}}$ IS AN EIGENVALUE FOR A^{-1}.

⑪ a) $c = 0$ b) $c = \pm 2$ c) $c \neq 0, 2, -2$

⑫ $x - 5y + z = 0$

⑬ LET $k = 2$, $l = 3$, $u = (1, 5)$; THEN

$\underline{(k+l)u} = 5(1,5) = (5,5)$ AND

$\underline{ku + lu} = 2(1,5) + 3(1,5) = (2,5) + (3,5) = (5,25)$,

SO $\underline{(k+l)u \neq ku + lu}$.

$\boxed{\text{OR}}$ $\vec{0} = (0,1)$, SO $u = (4,0)$ DOES NOT HAVE AN ADDITIVE INVERSE

SINCE $u + (x', y') = (4,0) + (x', y') = (4 + x', 0) \neq (0,1) = \vec{0}$.

(15) $[w]_B = \begin{bmatrix} -2 \\ 20 \end{bmatrix}$, so $w = \begin{bmatrix} 34 \\ 98 \end{bmatrix}$

(16) A) $A = \begin{bmatrix} 8 & 37 \\ -3 & -14 \end{bmatrix}$

B) $A = \begin{bmatrix} 66 & -19 \\ 200 & -57 \end{bmatrix}$

(17) SUPPOSE $\underline{c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 = \vec{0}}$.

1) $\underline{c_4 = 0}$, SINCE $c_4 \neq 0 \Rightarrow v_4 = -\frac{c_1}{c_4} v_1 - \frac{c_2}{c_4} v_2 - \frac{c_3}{c_4} v_3 \Rightarrow$

v_4 IS IN SPAN $\{v_1, v_2, v_3\}$, WHICH CONTRADICTS OUR ASSUMPTION,

2) THEN $\underline{c_1 v_1 + c_2 v_2 + c_3 v_3 = \vec{0}}$, SO $\underline{c_1 = 0,\ c_2 = 0,\ c_3 = 0}$ SINCE $\{v_1, v_2, v_3\}$ IS LI.

THEREFORE $\{v_1, v_2, v_3, v_4\}$ IS LI, SINCE $c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 = \vec{0}$ HAS ONLY THE TRIVIAL SOLUTION.

(18) $P = \begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix}$

(19) LET $S = \{v_1, \ldots, v_n\}$ AND $T = \{u_1, \ldots, u_m\}$ BE TWO BASES FOR V.

SINCE S IS LI AND T SPANS V, $\underline{n \leq m}$.

SINCE T IS LI AND S SPANS V, $\underline{m \leq n}$.

THEREFORE $\underline{m = n}$.

(20) $\text{PROJ}_W\, u = (3, 7, 5)$