PREREQUISITE REVIEW 9.1

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, solve for x.

1. $\frac{1}{9} + \frac{2}{3} + \frac{2}{9} = x$

2. $\frac{1}{3} + \frac{5}{12} + \frac{1}{8} + \frac{1}{12} + \frac{x}{24} = 1$

In Exercises 3–6, evaluate the expression.

3. $0(\frac{1}{16}) + 1(\frac{3}{16}) + 2(\frac{8}{16}) + 3(\frac{3}{16}) + 4(\frac{1}{16})$

4. $0(\frac{1}{12}) + 1(\frac{2}{12}) + 2(\frac{6}{12}) + 3(\frac{2}{12}) + 4(\frac{1}{12})$

5. $(0-1)^2(\frac{1}{4}) + (1-1)^2(\frac{1}{2}) + (2-1)^2(\frac{1}{4})$

6. $(0-2)^2(\frac{1}{12}) + (1-2)^2(\frac{2}{12}) + (2-2)^2(\frac{6}{12}) + (3-2)^2(\frac{2}{12}) + (4-2)^2(\frac{1}{12})$

In Exercises 7–10, write the fraction as a percent.

7. $\frac{3}{8}$ **8.** $\frac{9}{11}$

9. $\frac{13}{24}$ **10.** $\frac{112}{256}$

EXERCISES 9.1

In Exercises 1–4, list the elements in the specified set.

1. ***Coin Toss*** A coin is tossed three times.

(a) The sample space S

(b) The event A that at least two heads occur

(c) The event B that no more than one head occurs

2. ***Coin Toss*** A coin is tossed. If a head occurs, the coin is tossed again; otherwise, a die is tossed.

(a) The sample space S

(b) The event A that 4, 5, or 6 occurs on the die

(c) The event B that two heads occur

3. ***Random Selection*** An integer is selected from the set of all integers from 1 to 50 that are divisible by 3.

(a) The sample space S

(b) The event A that the integer is divisible by 12

(c) The event B that the integer is a perfect square

4. ***Poll*** Three people are asked their opinions on a political issue. They can answer "In favor" (I), "Opposed" (O), or "Undecided" (U).

(a) The sample space S

(b) The event A that at least two people are in favor

(c) The event B that no more than one person is opposed

5. ***Poll*** Three people have been nominated for president of a college class. From a small poll it is estimated that Jane has a probability of 0.29 of winning and Larry has a probability of 0.47. What is the probability of the third candidate winning the election?

6. ***Random Selection*** In a class of 72 students, 44 are girls and, of these, 12 are going to college. Of the 28 boys in the class, 9 are going to college. If a student is selected at random from the class, what is the probability that the person chosen is (a) going to college, (b) not going to college, and (c) a girl who is not going to college?

7. ***Quality Control*** A component of a spacecraft has both a main system and a backup system. The probability of at least one of the systems performing satisfactorily throughout the duration of the flight is 0.9855. What is the probability of both of them failing?

8. ***Random Selection*** A card is chosen at random from a standard 52-card deck of playing cards. What is the probability that the card will be black and a face card?

9. ***Coin Toss*** Two coins are tossed. A random variable assigns the number 0, 1, or 2 to each possible outcome, depending on the number of heads that turn up. Find the frequencies of 0, 1, and 2.

10. ***Coin Toss*** Four coins are tossed. A random variable assigns the number 0, 1, 2, 3, or 4 to each possible outcome, depending on the number of heads that turn up. Find the frequencies of 0, 1, 2, 3, and 4.

11. ***Exam*** Three students answer a true-false question on an examination. A random variable assigns the number 0, 1, 2, or 3 to each outcome, depending on the number of answers of *true* among the three students. Find the frequencies of 0, 1, 2, and 3.

12. ***Exam*** Four students answer a true-false question on an examination. A random variable assigns the number 0, 1, 2, 3, or 4 to each outcome, depending on the number of answers of *true* among the four students. Find the frequencies of 0, 1, 2, 3, and 4.

In Exercises 13–16, sketch a graph of the probability distribution and find the required probabilities.

13.

x	0	1	2	3	4
$P(x)$	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{6}{20}$	$\frac{4}{20}$

(a) $P(1 \le x \le 3)$

(b) $P(x \ge 2)$

14.

x	0	1	2	3	4
$P(x)$	$\frac{8}{20}$	$\frac{6}{20}$	$\frac{3}{20}$	$\frac{2}{20}$	$\frac{1}{20}$

(a) $P(x \le 2)$

(b) $P(x > 2)$

15.

x	0	1	2	3	4	5
$P(x)$	0.041	0.189	0.247	0.326	0.159	0.038

(a) $P(x \le 3)$

(b) $P(x > 3)$

16.

x	0	1	2	3
$P(x)$	0.027	0.189	0.441	0.343

(a) $P(1 \le x \le 2)$

(b) $P(x < 2)$

17. ***Biology*** Consider a couple that has four children. Assume that it is equally likely that each child is a girl or a boy.

(a) Complete the set to form the sample space consisting of 16 elements.

$$S = \{gggg, gggb, ggbg, . . .\}$$

(b) Complete the table, in which the random variable x is the number of girls in the family.

x	0	1	2	3	4
$P(x)$					

(c) Use the table in part (b) to sketch the graph of the probability distribution.

(d) Use the table in part (b) to find the probability of having at least one boy.

18. ***Die Toss*** Consider the experiment of tossing a 12-sided die twice.

(a) Complete the set to form the sample space of 144 elements. Note that each element is an ordered pair in which the entries are the numbers of points on the first and second tosses, respectively.

$$S = \{(1, 1), (1, 2), . . . , (2, 1), (2, 2), . . .\}$$

(b) Complete the table, in which the random variable x is the sum of the number of points.

x	2	3	4	5	6	7	8	9
$P(x)$								

x	10	11	12	13	14	15	16	17
$P(x)$								

x	18	19	20	21	22	23	24
$P(x)$							

(c) Use the table in part (b) to sketch the graph of the probability distribution.

(d) Use the table in part (b) to find $P(15 \le x \le 19)$.

19. ***Biology: Genetics*** Two parent plants, one with alleles RRYY (round, yellow seeds) and the other with alleles rryy (wrinkled, green seeds), are crossed to produce a first-generation RrYy (round, yellow seeds). When the first-generation plants (RrYy) are crossed among themselves, they produce offspring that take one of each type of allele from each plant. For example, an offspring could take R and y alleles from one parent and R and Y alleles from the other, making it RRYy (round, yellow seeds). Make a chart displaying all possibilities for alleles that the offspring could take on. Then list the probabilities of having the characteristics below. (*Source: Adapted from Levine/Miller Biology: Discovering Life, Second Edition*)

(a) Round, yellow seeds

(b) Wrinkled, yellow seeds

(c) Round, green seeds

(d) Wrinkled, green seeds

20. ***Experiment*** Use a computer to conduct the following experiment. The four letters A, R, S, and T are placed in a box. The letters are then chosen one at a time and placed in the order in which they were chosen. What is the probability that the letters spell an English word in the order they are chosen? Conduct this procedure 100 times. How many times did an English word occur?

In Exercises 21–24, find $E(x)$, $V(x)$, and σ for the given probability distribution.

21.

x	1	2	3	4	5
$P(x)$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{8}{16}$	$\frac{3}{16}$	$\frac{1}{16}$

22.

x	1	2	3	4	5
$P(x)$	$\frac{4}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

23.

x	-3	-1	0	3	5
$P(x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

24.

x	$-$\$5000	$-$\$2500	\$300
$P(x)$	0.008	0.052	0.940

In Exercises 25 and 26, find the mean and variance of the discrete random variable x.

25. ***Die Toss*** x is (a) the number of points when a four-sided die is tossed once and (b) the sum of the points when the four-sided die is tossed twice.

26. ***Coin Toss*** x is the number of heads when a coin is tossed four times.

27. ***Revenue*** A publishing company introduces a new weekly magazine that sells for \$2.95 on the newsstand. The marketing group of the company estimates that sales x in thousands will be approximated by the following probability function.

x	10	15	20	30	40
$P(x)$	0.25	0.30	0.25	0.15	0.05

(a) Find $E(x)$ and σ.

(b) Find the expected revenue.

28. ***Personal Income*** The probability distribution of the random variable x, the annual income of a family, in thousands of dollars, in a certain section of a large city, is shown in the table.

x	30	40	50	60	80
$P(x)$	0.10	0.20	0.50	0.15	0.05

Find $E(x)$ and σ.

29. ***Insurance*** An insurance company needs to determine the annual premium required to break even on fire protection policies with a face value of \$90,000. If x is the claim size on these policies and the analysis is restricted to the losses \$30,000, \$60,000, and \$90,000, then the probability distribution of x is shown in the table.

x	0	30,000	60,000	90,000
$P(x)$	0.995	0.0036	0.0011	0.0003

What premium should customers be charged for the company to break even?

30. ***Insurance*** An insurance company needs to determine the annual premium required to break even for collision protection for cars with a value of \$10,000. If x is the claim size on these policies and the analysis is restricted to the losses \$1000, \$5000, and \$10,000, then the probability distribution of x is shown in the table.

x	0	1000	5,000	10,000
$P(x)$	0.936	0.040	0.020	0.004

What premium should customers be charged for the company to break even?

Games of Chance If x is the net gain to a player in a game of chance, then $E(x)$ is usually negative. This value gives the average amount per game the player can expect to lose over the long run. In Exercises 31 and 32, find the expected net gain to the player for one play of the specified game.

31. In roulette, the wheel has the 38 numbers 00, 0, 1, 2, . . . , 34, 35, and 36, marked on equally spaced slots. If a player bets \$1 on a number and wins, then the player keeps the dollar and receives an additional \$35. Otherwise, the dollar is lost.

32. A service organization is selling \$2 raffle tickets as part of a fund-raising program. The first prize is a boat valued at \$2950, and the second prize is a camping tent valued at \$400. In addition to the first and second prizes, there are 25 \$20 gift certificates to be awarded. The number of tickets sold is 3000.

PREREQUISITE REVIEW 9.2

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, determine whether f is continuous and nonnegative on the given interval.

1. $f(x) = \frac{1}{x}$, $[1, 4]$

2. $f(x) = x^2 - 1$, $[0, 1]$

3. $f(x) = 3 - x$, $[1, 5]$

4. $f(x) = e^{-x}$, $[0, 1]$

In Exercises 5–10, evaluate the definite integral.

5. $\int_0^4 \frac{1}{4}\, dx$

6. $\int_1^3 \frac{1}{4}\, dx$

7. $\int_0^2 \frac{2 - x}{2}\, dx$

8. $\int_1^2 \frac{2 - x}{2}\, dx$

9. $\int_0^\infty 0.4e^{-0.4t}\, dt$

10. $\int_0^\infty 3e^{-3t}\, dt$

EXERCISES 9.2

In Exercises 1–14, use a graphing utility to graph the function. Then verify that f is a probability density function over the given interval.

1. $f(x) = \frac{1}{8}$, $[0, 8]$

2. $f(x) = \frac{1}{5}$, $[0, 5]$

3. $f(x) = \frac{4 - x}{8}$, $[0, 4]$

4. $f(x) = \frac{x}{18}$, $[0, 6]$

5. $f(x) = 6x(1 - x)$, $[0, 1]$

6. $f(x) = \frac{x(6 - x)}{36}$, $[0, 6]$

7. $f(x) = \frac{1}{5}e^{-x/5}$, $[0, \infty)$

8. $f(x) = \frac{1}{6}e^{-x/6}$, $[0, \infty)$

9. $f(x) = \frac{3x}{8}\sqrt{4 - x^2}$, $[0, 2]$

10. $f(x) = 12x^2(1 - x)$, $[0, 1]$

11. $f(x) = \frac{4}{27}x^2(3 - x)$, $[0, 3]$

12. $f(x) = \frac{2}{9}x(3 - x)$, $[0, 3]$

13. $f(x) = \frac{1}{3}e^{-x/3}$, $[0, \infty)$

14. $f(x) = \frac{1}{4}$, $[8, 12]$

In Exercises 15–20, find the constant k such that the function f is a probability density function over the given interval.

15. $f(x) = kx$, $[1, 4]$

16. $f(x) = kx^3$, $[0, 4]$

17. $f(x) = k(4 - x^2)$, $[-2, 2]$

18. $f(x) = k\sqrt{x}(1 - x)$, $[0, 1]$

19. $f(x) = ke^{-x/2}$, $[0, \infty)$

20. $f(x) = \frac{k}{b - a}$, $[a, b]$

In Exercises 21–26, sketch the graph of the probability density function over the indicated interval and find the indicated probabilities.

21. $f(x) = \frac{1}{10}$, $[0, 10]$

(a) $P(0 < x < 6)$ (b) $P(4 < x < 6)$

(c) $P(8 < x < 10)$ (d) $P(x \geq 2)$

22. $f(x) = \frac{x}{50}$, $[0, 10]$

(a) $P(0 < x < 6)$ (b) $P(4 < x < 6)$

(c) $P(8 < x < 10)$ (d) $P(x \geq 2)$

23. $f(x) = \frac{3}{16}\sqrt{x}$, $[0, 4]$

(a) $P(0 < x < 2)$ (b) $P(2 < x < 4)$

(c) $P(1 < x < 3)$ (d) $P(x \le 3)$

24. $f(x) = \dfrac{5}{4(x+1)^2}$, $[0, 4]$

(a) $P(0 < x < 2)$ (b) $P(2 < x < 4)$

(c) $P(1 < x < 3)$ (d) $P(x \le 3)$

25. $f(t) = \frac{1}{3}e^{-t/3}$, $[0, \infty)$

(a) $P(t < 2)$ (b) $P(t \ge 2)$

(c) $P(1 < t < 4)$ (d) $P(t = 3)$

26. $f(t) = \frac{3}{256}(16 - t^2)$, $[-4, 4]$

(a) $P(t < -2)$ (b) $P(t > 2)$

(c) $P(-1 < t < 1)$ (d) $P(t > -2)$

27. ***Waiting Time*** Buses arrive and depart from a college every 30 minutes. The probability density function for the waiting time t (in minutes) for a person arriving at the bus stop is

$$f(t) = \tfrac{1}{30}, \quad [0, 30].$$

Find the probabilities that the person will wait (a) no more than 5 minutes and (b) at least 18 minutes.

28. ***Learning Theory*** The time t (in hours) required for a new employee successfully to learn to operate a machine in a manufacturing process is described by the probability density function

$$f(t) = \tfrac{5}{324}t\sqrt{9 - t}, \quad [0, 9].$$

Find the probabilities that a new employee will learn to operate the machine (a) in less than 3 hours and (b) in more than 4 hours but less than 8 hours.

In Exercises 29–32, use a symbolic integration utility to find the required probabilities using the *exponential density function*

$$f(t) = \frac{1}{\lambda}e^{-t/\lambda}, \quad [0, \infty).$$

29. ***Waiting Time*** The waiting time (in minutes) for service at the checkout at a grocery store is exponentially distributed with $\lambda = 3$. Find the probabilities of waiting (a) less than 2 minutes, (b) more than 2 minutes but less than 4 minutes, and (c) at least 2 minutes.

30. ***Useful Life*** The lifetime (in years) of a battery is exponentially distributed with $\lambda = 5$. Find the probabilities that the lifetime of a given battery will be (a) less than 6 years, (b) more than 2 years but less than 6 years, and (c) more than 8 years.

31. ***Waiting Time*** The length of time (in hours) required to unload trucks at a depot is exponentially distributed with $\lambda = \frac{3}{4}$. What proportion of the trucks can be unloaded in less than 1 hour?

32. ***Useful Life*** The time (in years) until failure of a component in a machine is exponentially distributed with $\lambda = 3.5$. A manufacturer has a large number of these machines and plans to replace the components in all the machines during regularly scheduled maintenance periods. How much time should elapse between maintenance periods if at least 90% of the components are to remain working throughout the period?

33. ***Demand*** The weekly demand x (in tons) for a certain product is a continuous random variable with the density function

$$f(x) = \tfrac{1}{36}xe^{-x/6}, \quad [0, \infty).$$

Find the probabilities.

(a) $P(x < 6)$

(b) $P(6 < x < 12)$

(c) $P(x > 12) = 1 - P(x \le 12)$

34. ***Demand*** Given the conditions of Exercise 33, determine the number of tons that should be ordered each week so that the demand can be met for 90% of the weeks.

35. ***Meteorology*** A meteorologist predicts that the amount of rainfall (in inches) expected for a certain coastal community during a hurricane has the probability density function

$$f(x) = \frac{\pi}{30}\sin\frac{\pi x}{15}, \quad 0 \le x \le 15.$$

Find and interpret the probabilities.

(a) $P(0 \le x \le 10)$ (b) $P(10 \le x \le 15)$

(c) $P(0 \le x < 5)$ (d) $P(12 \le x \le 15)$

36. ***Metallurgy*** The probability density function for the percent of iron in ore samples taken from a certain region is

$$f(x) = \tfrac{1155}{32}x^3(1 - x)^{3/2}, \quad [0, 1].$$

Find the probabilities that a sample will contain (a) from 0% to 25% iron and (b) from 50% to 100% iron.

37. ***Labor Force*** The probability density function for women in the U.S. civilian labor force in 2002, ages 16 and over, is

$$f(x) = 4.7 + 0.61x - 4.15\ln x - 5.14/x, \quad 1 \le x \le 5$$

where $x = 1$ represents 16 to 24 years, $x = 2$ represents 25 to 44 years, $x = 3$ represents 45 to 54 years, $x = 4$ represents 55 to 64 years, and $x = 5$ represents 65 years and over. Find the probabilities that a woman in the labor force is (a) between the ages of 16 and 64 and (b) between the ages of 25 and 54. (Source: U.S. Bureau of Labor Statistics)

38. ***Coin Toss*** The probability of obtaining 49, 50, or 51 heads when a fair coin is tossed 100 times is

$$P(49 \le x \le 51) \approx \int_{48.5}^{51.5} \frac{1}{5\sqrt{2\pi}}e^{-(x-50)^2/50}\,dx.$$

Use a computer or graphing utility and Simpson's Rule (with $n = 12$) to approximate this integral.

mean on of s that ween one tribu- one % of st all

PREREQUISITE REVIEW 9.3

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, solve for m.

1. $\int_0^m \frac{1}{10}\,dx = 0.5$
2. $\int_0^m \frac{1}{16}\,dx = 0.5$
3. $\int_0^m \frac{1}{3}e^{-t/3}\,dt = 0.5$
4. $\int_0^m \frac{1}{9}e^{-t/9}\,dt = 0.5$

In Exercises 5–8, evaluate the definite integral.

5. $\int_0^2 \frac{x^2}{2}\,dx$
6. $\int_1^2 x(4-2x)\,dx$
7. $\int_2^5 x^2\left(\frac{1}{3}\right)dx - \left(\frac{7}{2}\right)^2$
8. $\int_2^4 x^2\left(\frac{4-x}{2}\right)dx - \left(\frac{8}{3}\right)^2$

In Exercises 9 and 10, find the indicated probability using the given probability density function.

9. $f(x) = \frac{1}{8}$, $[0, 8]$
   (a) $P(x \le 2)$,
   (b) $P(3 < x < 7)$
10. $f(x) = 6x - 6x^2$, $[0, 1]$
   (a) $P\left(x \le \frac{1}{2}\right)$,
   (b) $P\left(\frac{1}{4} \le x \le \frac{3}{4}\right)$

EXERCISES 9.3

In Exercises 1–6, use the given probability density function over the indicated interval to find (a) the mean, (b) the variance, and (c) the standard deviation of the random variable. Sketch the graph of the density function and locate the mean on the graph.

1. $f(x) = \frac{1}{8}$, $[0, 8]$
2. $f(x) = \frac{1}{4}$, $[0, 4]$
3. $f(t) = \frac{t}{18}$, $[0, 6]$
4. $f(x) = \frac{4}{3x^2}$, $[1, 4]$
5. $f(x) = \frac{5}{2}x^{3/2}$, $[0, 1]$
6. $f(x) = \frac{3}{16}\sqrt{4-x}$, $[0, 4]$

In Exercises 7–10, use a graphing utility to graph the function and approximate the mean. Then find the mean analytically. Compare your results.

7. $f(x) = 6x(1-x)$, $[0, 1]$
8. $f(x) = \frac{3}{32}x(4-x)$, $[0, 4]$
9. $f(x) = \frac{4}{3(x+1)^2}$, $[0, 3]$
10. $f(x) = \frac{1}{18}\sqrt{9-x}$, $[0, 9]$

In Exercises 11 and 12, find the median of the exponential probability density function.

11. $f(t) = \frac{1}{9}e^{-t/9}$, $[0, \infty)$
12. $f(t) = \frac{2}{5}e^{-2t/5}$, $[0, \infty)$

In Exercises 13–18, identify the probability density function. Then find the mean, variance, and standard deviation without integrating.

13. $f(x) = \frac{1}{10}$, $[0, 10]$
14. $f(x) = \frac{1}{9}$, $[0, 9]$
15. $f(x) = \frac{1}{8}e^{-x/8}$, $[0, \infty)$
16. $f(x) = \frac{5}{3}e^{-5x/3}$, $[0, \infty)$
17. $f(x) = \frac{1}{11\sqrt{2\pi}}e^{-(x-100)^2/242}$, $(-\infty, \infty)$
18. $f(x) = \frac{1}{6\sqrt{2\pi}}e^{-(x-30)^2/72}$, $(-\infty, \infty)$

In Exercises 19–24, use a symbolic integration utility to find the mean, standard deviation, and given probability.

Function	*Probability*
19. $f(x) = \dfrac{1}{\sqrt{2\pi}}e^{-x^2/2}$	$P(0 \le x \le 0.85)$
20. $f(x) = \dfrac{1}{\sqrt{2\pi}}e^{-x^2/2}$	$P(-1.21 \le x \le 1.21)$
21. $f(x) = \frac{1}{6}e^{-x/6}$	$P(x \ge 2.23)$
22. $f(x) = \frac{3}{4}e^{-3x/4}$	$P(x \ge 0.27)$
23. $f(x) = \dfrac{1}{2\sqrt{2\pi}}e^{-(x-8)^2/8}$	$P(3 \le x \le 13)$
24. $f(x) = \dfrac{1}{1.5\sqrt{2\pi}}e^{-(x-2)^2/4.5}$	$P(-2.5 \le x \le 2.5)$

25. Let x be a random variable that is normally distributed with a mean of 60 and a standard deviation of 12. Find the required probabilities using a symbolic integration utility.

(a) $P(x > 64)$

(b) $P(x > 70)$

(c) $P(x < 70)$

(d) $P(33 < x < 65)$

26. Repeat Exercise 25, with a mean of 75 and a standard deviation of 12, for the following probabilities.

(a) $P(x > 69)$

(b) $P(x < 99)$

(c) $P(x < 57)$

(d) $P(63 < x < 78)$

27. ***Transportation*** The arrival time t of a bus at a bus stop is uniformly distributed between 10:00 A.M. and 10:10 A.M.

(a) Find the mean and standard deviation of the random variable t.

(b) What is the probability that you will miss the bus if you arrive at the bus stop at 10:03 A.M.?

28. ***Transportation*** Repeat Exercise 27 for a bus that arrives between 10:00 A.M. and 10:05 A.M.

29. ***Useful Life*** The time t until failure of an appliance is exponentially distributed with a mean of 2 years.

(a) Find the probability density function for the random variable t.

(b) Find the probability that the appliance will fail in less than 1 year.

30. ***Useful Life*** The lifetime of a battery is normally distributed with a mean of 400 hours and a standard deviation of 24 hours. You purchased one of the batteries, and its useful life was 340 hours.

(a) How far, in standard deviations, did the useful life of your battery fall short of the expected life?

(b) What percent of all other batteries of this type have useful lives that exceed yours?

31. ***Waiting Time*** The waiting time t for service in a store is exponentially distributed with a mean of 5 minutes.

(a) Find the probability density function for the random variable t.

(b) Find the probability that t is within one standard deviation of the mean.

32. ***Service Time*** The service time t for a customer in a store is exponentially distributed with a mean of 3.5 minutes.

(a) Find the probability density function for the random variable t.

(b) Find the probability that t is within one standard deviation of the mean.

33. ***Education*** The scores on a national exam are normally distributed with a mean of 150 and a standard deviation of 16. You scored 174 on the exam.

(a) How far, in standard deviations, did your score exceed the national mean?

(b) What percent of those who took the exam had scores lower than yours?

34. ***Education*** The scores on a qualifying exam for entrance into a post secondary school are normally distributed with a mean of 120 and a standard deviation of 10.5. To qualify for admittance, the candidates must score in the top 10%. Find the lowest possible score to qualify.

35. ***Demand*** The daily demand x for a certain product (in hundreds of pounds) is a random variable with the probability density function

$$f(x) = \tfrac{1}{36}x(6 - x), \quad [0, 6].$$

(a) Determine the expected value and the standard deviation of demand.

(b) Determine the median of the random variable.

(c) Find the probability that x is within one standard deviation of the mean.

36. ***Demand*** Repeat Exercise 35 for a probability density function of

$$f(x) = \tfrac{3}{256}(x - 2)(10 - x), \quad [2, 10].$$

37. ***Learning Theory*** The percent recall x in a learning experiment is a random variable with the probability density function

$$f(x) = \tfrac{15}{4}x\sqrt{1 - x}, \quad [0, 1].$$

Determine the mean and variance of the random variable x.

38. ***Metallurgy*** The percent of iron x in samples of ore is a random variable with the probability density function

$$f(x) = \tfrac{1155}{32}x^3(1-x)^{3/2}, \quad [0, 1].$$

Determine the expected percent of iron in each ore sample.

39. ***Demand*** The daily demand x for a certain product (in thousands of units) is a random variable with the probability density function

$$f(x) = \tfrac{1}{25}xe^{-x/5}, \quad [0, \infty).$$

(a) Determine the expected daily demand.

(b) Find $P(x \le 4)$.

40. ***Medicine*** The time t (in days) until recovery after a certain medical procedure is a random variable with the probability density function

$$f(t) = \frac{1}{2\sqrt{t-2}}, \quad [3, 6].$$

(a) Find the probability that a patient selected at random will take more than 4 days to recover.

(b) Determine the expected time for recovery.

In Exercises 41–46, find the mean and median.

41. $f(x) = \frac{1}{11}$	$[0, 11]$
42. $f(x) = 0.04$	$[0, 25]$
43. $f(x) = 4(1-2x)$	$\left[0, \frac{1}{2}\right]$
44. $f(x) = \frac{4}{3} - \frac{2}{3}x$	$[0, 1]$
45. $f(x) = \frac{1}{5}e^{-x/5}$	$[0, \infty)$
46. $f(x) = \frac{2}{3}e^{-2x/3}$	$[0, \infty)$

47. ***Cost*** The daily cost of electricity x in a city is a random variable with the probability density function

$$f(x) = 0.28e^{-0.28x}, \quad 0 \le x < \infty.$$

Find the median daily cost of electricity.

48. ***Consumer Trends*** The number of coupons used by a customer in a grocery store is a random variable with the probability density function

$$f(x) = \frac{2x+1}{10}, \quad 1 \le x \le 2.$$

Find the expected number of coupons a customer will use.

49. ***Demand*** The daily demand x for water (in millions of gallons) in a town is a random variable with the probability density function

$$f(x) = \tfrac{1}{9}xe^{-x/3}, \quad [0, \infty).$$

(a) Determine the expected value and the standard deviation of the demand.

(b) Find the probability that the demand is greater than 4 million gallons on a given day.

50. ***Useful Life*** The lifetime of an electrical component is normally distributed with a mean of 5.3 years and a standard deviation of 0.8 year. How long should this component be guaranteed if the producer does not want to replace any more than 10% of the components during the time covered by the guarantee?

51. ***Manufacturing*** An automatic filling machine fills cans so that the weights are normally distributed with a mean of μ and a standard deviation of σ. The value of μ can be controlled by settings on the machine, but σ depends on the precision and design of the machine. For a particular substance, $\sigma = 0.15$ ounce. If 12-ounce cans are being filled, determine the setting for μ such that no more than 5% of the cans weigh less than the stated weight.

52. ***Useful Life*** A storage battery has an expected lifetime of 4.5 years with a standard deviation of 0.5 year. Assume that the useful lives of these batteries are normally distributed. Use a computer or graphing utility and Simpson's Rule (with $n = 12$) to approximate the probability that a given battery will last for 4 to 5 years.

53. ***Wages*** The employees of a large corporation are paid an average wage of \$12.30 per hour with a standard deviation of \$1.50. Assume that these wages are normally distributed. Use a computer or graphing utility and Simpson's Rule (with $n = 10$) to approximate the percent of employees that earn hourly wages of \$9.00 to \$12.00.

54. ***Medical Science*** A medical research team has determined that for a group of 500 females, the length of pregnancy from conception to birth varies according to an approximately normal distribution with a mean of 266 days and a standard deviation of 16 days.

(a) Use a graphing utility to graph the distribution.

(b) Use a symbolic integration utility to approximate the probability that a pregnancy will last from 240 days to 280 days.

55. ***Education*** In 2003, the scores for the ACT Test could be modeled by a normal probability density function with a mean of $\mu = 20.8$ and a standard deviation of $\sigma = 4.8$. *(Source: ACT, Inc.)*

(a) Use a graphing utility to graph the distribution.

(b) Use a symbolic integration utility to approximate the probability that a person who took the ACT scored between 24 and 36.

56. ***Intelligence Quotient*** The IQs of students in a school are normally distributed with a mean of 110 and a standard deviation of 10. Use a symbolic integration utility to find the probability that a student selected at random will have an IQ within one standard deviation of the mean.

9 CHAPTER REVIEW EXERCISES

In Exercises 1–4, describe the sample space of the experiment.

1. A month of the year is chosen for vacation.
2. A letter from the word *calculus* is selected.
3. A student must answer three questions from a selection of four essay questions.
4. A winner in a game show must choose two out of five prizes.
5. ***Lottery*** Three numbers are drawn in a lottery. Each number is a digit from 0 to 9. Find the sample space giving the number of 7's drawn.
6. ***Quality Control*** As cans of soft drink are filled on the production line, four are randomly selected and labeled with an "S" if the weight is satisfactory or with a "U" if the weight is unsatisfactory. Find the sample space giving the satisfactory/unsatisfactory classification of the four cans in the selected group.

In Exercises 7 and 8, complete the table to form the frequency distribution of the random variable x. Then construct a bar graph to represent the result.

7. A computer randomly selects a three-digit bar code. Each digit can be 0 or 1, and x is the number of 1's in the bar code.

x	0	1	2	3
$n(x)$				

8. A cat has a litter of four kittens. Let x represent the number of male kittens.

x	0	1	2	3	4
$n(x)$					

In Exercises 9 and 10, sketch a graph of the given probability distribution and find the required probabilities.

9.

x	1	2	3	4	5
$P(x)$	$\frac{1}{18}$	$\frac{7}{18}$	$\frac{5}{18}$	$\frac{3}{18}$	$\frac{2}{18}$

(a) $P(2 \le x \le 4)$

(b) $P(x \ge 3)$

10.

x	-2	-1	1	3	5
$P(x)$	$\frac{1}{11}$	$\frac{2}{11}$	$\frac{4}{11}$	$\frac{3}{11}$	$\frac{1}{11}$

(a) $P(x < 0)$

(b) $P(x > 1)$

11. ***Dice Toss*** Consider an experiment in which two six-sided dice are tossed. Find the indicated probabilities.
    (a) The probability that the total is 8
    (b) The probability that the total is greater than 4
    (c) The probability that doubles are thrown
    (d) The probability of getting double 6's
12. ***Random Selection*** Consider an experiment in which one card is randomly selected from a standard deck of 52 playing cards. Find the probabilities of
    (a) selecting a face card.
    (b) selecting a card that is not a face card.
    (c) selecting a black card that is not a face card.
    (d) selecting a card whose value is 6 or less.
13. ***Education*** An instructor gave a 25-point quiz to 52 students. Use the frequency distribution shown below to find the mean quiz score.

Score	9	10	11	12	13	14	15	16	17
Frequency	1	0	1	0	0	0	3	4	7

Score	18	19	20	21	22	23	24	25
Frequency	3	0	9	11	6	3	0	4

14. ***Cost Increases*** A pharmaceutical company uses three different chemicals, A, B, and C, to create a nutritional supplement. The table shown below gives the cost and the percent increase of the cost of each of the three chemicals. Find the mean percent increase of the three chemicals.

Chemical	*Percent Increase*	*Cost of Materials*
A	8%	\$650
B	23%	\$375
C	16%	\$800

15. ***Revenue*** A publishing company introduces a new weekly newspaper that sells for 75 cents. The marketing group of the company estimates that sales x (in thousands) will be approximated by the probability function shown in the table.

x	10	15	20	30	40
$P(x)$	0.10	0.20	0.50	0.15	0.05

(a) Find $E(x)$.

(b) Find the expected revenue.

16. ***Games of Chance*** A service organization is selling $3 raffle tickets as part of a fund-raising program. The first and second prizes are $2000 and $1000, respectively. In addition to the first and second prizes, there are 50 $20 gift certificates to be awarded. The number of tickets sold is 2000. Find the expected net gain to the player when one ticket is purchased.

17. ***Sales*** A company sells five different models of personal computers. During one month the sales for the five models were as shown.

Model 1	24 sold at $1200 each
Model 2	12 sold at $1500 each
Model 3	35 sold at $2000 each
Model 4	5 sold at $2200 each
Model 5	4 sold at $3000 each

Find the variance and standard deviation of the prices.

18. ***Inventory*** A school buys multiple copies of five different mathematics textbooks. The quantities and prices per book are as shown.

Geometry	12 copies at $54 each
Algebra	45 copies at $45 each
Precalculus	25 copies at $52 each
Calculus	20 copies at $60 each
Statistics	15 copies at $65 each

Find the variance and standard deviation of the prices.

19. ***Consumer Trends*** A random survey of households recorded the number of cars per household. The results of the survey are shown in the table.

x	0	1	2	3	4	5
$P(x)$	0.10	0.28	0.39	0.17	0.04	0.02

Find the variance and standard deviation of x.

20. ***Vital Statistics*** The probability distribution for the numbers of children in a sample of families is shown in the table.

x	0	1	2	3	4
$P(x)$	0.12	0.31	0.43	0.12	0.02

Find the variance and standard deviation of x.

In Exercises 21–24, use a graphing utility to graph the function. Then verify that f is a probability density function.

21. $f(x) = \frac{1}{8}(4 - x), \quad [0, 4]$

22. $f(x) = \frac{3}{4}x^2(2 - x), \quad [0, 2]$

23. $f(x) = \dfrac{1}{4\sqrt{x}}, \quad [1, 9]$

24. $f(x) = 8.75x^{3/2}(1 - x), \quad [0, 1]$

In Exercises 25–28, find the indicated probability for the probability density function.

25. $f(x) = \frac{1}{50}(10 - x), \quad [0, 10]$

$P(0 < x < 2)$

26. $f(x) = \frac{1}{36}(9 - x^2), \quad [-3, 3]$

$P(-1 < x < 2)$

27. $f(x) = \dfrac{2}{(x + 1)^2}, \quad [0, 1]$

$P\left(0 < x < \frac{1}{2}\right)$

28. $f(x) = \dfrac{3}{128}\sqrt{x}, \quad [0, 16]$

$P(4 < x < 9)$

29. ***Waiting Time*** Buses arrive and depart from a college every 20 minutes. The probability density function for the waiting time t (in minutes) for a person arriving at the bus stop is

$$f(t) = \frac{1}{20}, \quad [0, 20].$$

Find the probabilities that the person will wait (a) no more than 10 minutes and (b) at least 15 minutes.

30. ***Learning Theory*** The time t (in hours) required for a new employee successfully to learn to operate a machine in a manufacturing process is described by the probability density function

$$f(t) = \frac{5}{324}t\sqrt{9 - t}, \quad [0, 9].$$

Find the probability that a new employee will learn to operate the machine in less than 4 hours.

31. ***Medicine*** The time t (in days) until recovery after a certain medical procedure is a random variable with the probability density function

$$f(t) = \frac{1}{4\sqrt{t - 4}}, \quad [5, 13].$$

Find the probability that a patient selected at random will take more than 8 days to recover.

32. *Vit*
nun
Stat
whe
peo
four
the
peo

In Exerci
function.

33. $f(x)$

34. $f(x)$

35. $f(x)$

36. $f(x)$

In Exercis
the proba

37. $f(x)$

38. $f(x)$

39. $f(x)$

40. $f(x)$

In Exercis
function.

41. $f(x) =$

42. $f(x) =$

43. $f(x) =$

44. $f(x) =$

45. *Waiti*
at the
uted w
$f(t$
Find th
(b) mo

46. *Usefu*
unit is
functic
$f(t)$
Find th
perforr

32. ***Vital Statistics*** The probability density function for the number of people in households in 2002 in the United States is

$$f(x) = 1.77 - 0.15x - \frac{4.8}{x} + 9.4e^{-x}, \quad 1 \le x \le 5$$

where $x = 1$ represents one person, $x = 2$ represents two people, $x = 3$ represents three people, $x = 4$ represents four people, and $x = 5$ represents five or more people. Find the probabilities that a household has (a) from two to four people and (b) three or more people. *(Source: U.S. Census Bureau)*

In Exercises 33–36, find the mean of the probability density function.

33. $f(x) = \frac{1}{5}, \quad [0, 5]$

34. $f(x) = \frac{8 - x}{32}, \quad [0, 8]$

35. $f(x) = \frac{1}{6}e^{-x/6}, \quad [0, \infty)$

36. $f(x) = 0.3e^{-0.3x}, \quad [0, \infty)$

In Exercises 37–40, find the variance and standard deviation of the probability density function.

37. $f(x) = \frac{2}{9}x(3 - x), \quad [0, 3]$

38. $f(x) = \frac{3}{16}\sqrt{x}, \quad [0, 4]$

39. $f(x) = \frac{1}{2}e^{-x/2}, \quad [0, \infty)$

40. $f(x) = 0.8e^{-0.8x}, \quad [0, \infty)$

In Exercises 41–44, find the median of the probability density function.

41. $f(x) = 6x(1 - x), \quad [0, 1]$

42. $f(x) = 12x^2(1 - x), \quad [0, 1]$

43. $f(x) = 0.25e^{-x/4}, \quad [0, \infty)$

44. $f(x) = \frac{5}{6}e^{-5x/6}, \quad [0, \infty)$

45. ***Waiting Time*** The waiting time t (in minutes) for service at the checkout at a grocery store is exponentially distributed with the probability density function

$$f(t) = \tfrac{1}{3}e^{-t/3}, \quad [0, \infty).$$

Find the probabilities of waiting (a) less than 2 minutes and (b) more than 2 minutes but less than 4 minutes.

46. ***Useful Life*** The lifetime t (in hours) of a mechanical unit is exponentially distributed with the following density function.

$$f(t) = \tfrac{1}{350}e^{-t/350}, \quad [0, \infty).$$

Find the probability that a given unit chosen at random will perform satisfactorily for more than 400 hours.

47. ***Botany*** In a botany experiment, plants are grown in a nutrient solution. The heights of the plants are found to be normally distributed with a mean of 42 centimeters and a standard deviation of 3 centimeters. Find the probability that a plant in the experiment is at least 50 centimeters tall.

48. ***Wages*** The hourly wages for the workers at a certain company are normally distributed with a mean of \$14.50 and a standard deviation of \$1.40.

(a) What percent of the workers receive hourly wages from \$13 to \$15, inclusive?

(b) The highest 10% of the hourly wages are greater than what amount?

49. ***Meteorology*** The monthly rainfall x in a certain state is normally distributed with a mean of 3.75 inches and a standard deviation of 0.5 inch. Use a computer or a graphing utility and Simpson's Rule (with $n = 12$) to approximate the probability that in a randomly selected month the rainfall is between 3.5 and 4 inches.

50. ***Demand*** The weekly demand x (in tons) for a certain product is a continuous random variable with the density function

$$f(x) = \tfrac{1}{25}xe^{-x/5}, \quad [0, \infty).$$

Use a symbolic integration utility to find the probabilities.

(a) $P(x < 5)$

(b) $P(5 < x < 10)$

(c) $P(x > 10) = 1 - P(x \le 10)$

51. ***Chemistry: Hydrogen Orbitals*** In chemistry, the probability of finding an electron at a particular position is greatest close to the nucleus and drops off rapidly as the distance from the nucleus increases. The graph displays the probability of finding the electron at points along a line drawn from the nucleus outward in any direction for the hydrogen $1s$ orbital. Make a sketch of this graph, and add to your sketch an indication of where you think the median might be. *(Source: Adapted from Zumdahl, Chemistry, Sixth Edition)*