(b) $\frac{dR}{dx} = -2.86x + 13.8$;

10.94, 8.08, 5.22, 2.36, −0.50, −3.36;

The model is a fairly good estimate.

(c) About 5 units of output: (4.83, 35.29)

SAMPLE POST-GRAD EXAM QUESTIONS

(page 256)

1. b **2.** a **3.** b **4.** d **5.** b

CHAPTER 4

SECTION 4.1 *(page 262)*

Prerequisite Review

1. Horizontal shift to the left two units
2. Reflection about the x-axis
3. Vertical shift down one unit
4. Reflection about the y-axis
5. Horizontal shift to the right one unit
6. Vertical shift up two units
7. Nonremovable discontinuity at $x = -4$
8. Continuous on $(-\infty, \infty)$
9. Discontinuous at $x = \pm 1$
10. Continuous on $(-\infty, \infty)$
11. 5 **12.** $\frac{4}{3}$ **13.** $-9, 1$ **14.** $2 \pm 2\sqrt{2}$
15. $1, -5$ **16.** $\frac{1}{2}, 1$

1. (a) 625 (b) 9 (c) $16\sqrt{2}$
(d) 9 (e) 125 (f) 4

3. (a) 3125 (b) $\frac{1}{5}$ (c) 625 (d) $\frac{1}{125}$

5. (a) $\frac{1}{5}$ (b) 27 (c) 5 (d) 4096

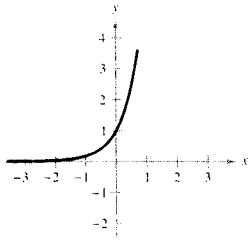
7. (a) 4 (b) $\frac{\sqrt{2}}{2} \approx 0.707$ (c) $\frac{1}{8}$ (d) $\frac{\sqrt{2}}{8} \approx 0.177$

9. (a) 0.907 (b) 348.912 (c) 1.796 (d) 1.308

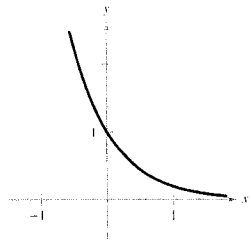
11. 4 **13.** −2 **15.** 2 **17.** 16 **19.** e

20. c **21.** a **22.** f **23.** d **24.** b

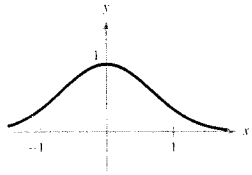
25.



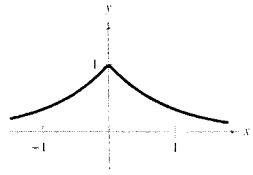
27.



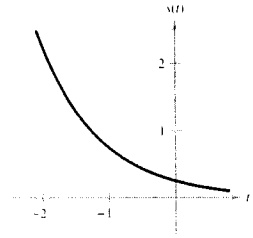
29.



31.



33.

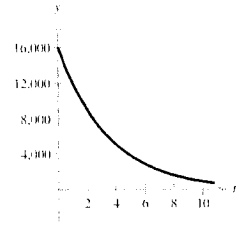


35. (a) $P(16) \approx 303.15$ million

(b) $P(22) \approx 325.25$ million

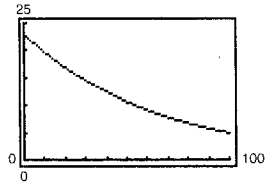
37. (a) $V(5) \approx \$80{,}634.95$ (b) $V(20) \approx \$161{,}269.89$

39. $V(t) = 16{,}000\left(\frac{3}{4}\right)^t$



$V(4) = 16{,}000\left(\frac{3}{4}\right)^4 \approx \$5062.50$

41. (a)



(b) $y(75) \approx 7.24$ grams (c) $y = 1$ for $t \approx 203.56$ years

SECTION 4.2 *(page 270)*

Prerequisite Review

1. Continuous on $(-\infty, \infty)$
2. Discontinuous for $x = \pm 2$
3. Discontinuous for $x = \pm\sqrt{3}$
4. Removable discontinuity at $x = 4$

5. 0 **6.** 0 **7.** 4 **8.** $\frac{1}{2}$ **9.** $\frac{3}{2}$
10. 6 **11.** 0 **12.** 0

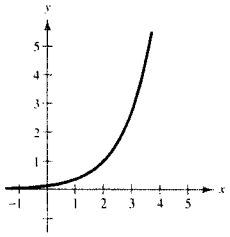
1. (a) e^7 (b) e^{12} (c) $\dfrac{1}{e^6}$ (d) 1

3. (a) e^5 (b) $e^{5/2}$ (c) e^6 (d) e^7

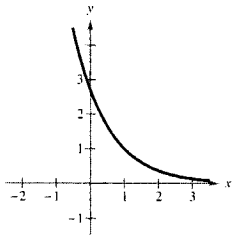
5. $-\frac{1}{3}$ **7.** 9 **9.** $\pm e$ **11.** $\sqrt[3]{3}e$ **13.** f

14. e **15.** d **16.** b **17.** c **18.** a

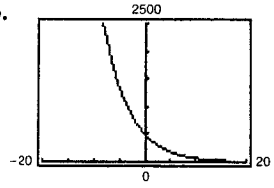
19.



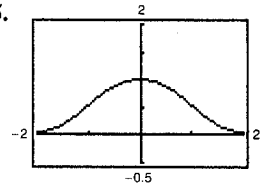
21.



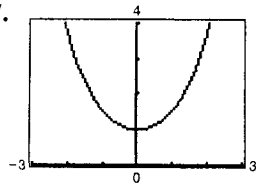
23.



25.



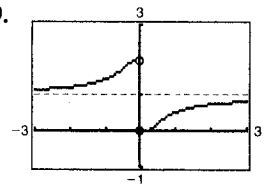
27.



No horizontal asymptotes

Continuous on the entire real number line

29.



Horizontal asymptote: $y = 1$

Discontinuous at $x = 0$

31.

n	1	2	4	12
A	1343.92	1346.86	1348.35	1349.35

n	365	Continuous compounding
A	1349.84	1349.86

33.

n	1	2	4	12
A	3262.04	3290.66	3305.28	3315.15

n	365	Continuous compounding
A	3319.95	3320.12

35.

t	1	10	20
P	96,078.94	67,032.00	44,932.90

t	30	40	50
P	30,119.42	20,189.65	13,533.53

37.

t	1	10	20
P	95,132.82	60,716.10	36,864.45

t	30	40	50
P	22,382.66	13,589.88	8251.24

39. (a) 9% (b) 9.2% (c) 9.31% (d) 9.38%

41. \$12,500 **43.** \$8751.92

45. (a) \$849.53 (b) \$421.12

$\lim_{x\to\infty} p = 0$

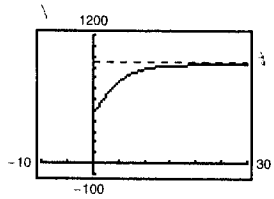
47. (a) 0.1535 (b) 0.4866 (c) 0.8111

49. (a) 1995: \$4510.69 million; 2000: \$5719.90 million
2003: \$6595.92 million

(b) Yes. There is a positive correlation between sales and time in years.

(c) 2011

51. (a)



(b) Yes, $\lim_{t\to\infty} \dfrac{925}{1 + e^{-0.3t}} = 925$

(c) $\lim_{t\to\infty} \dfrac{1000}{1 + e^{-0.3t}} = 1000$

Models similar to this logistic growth model where $y = \dfrac{a}{1 + be^{-ct}}$ have a limit of a as $t \to \infty$.

53. (a) 0.731 (b) 11 (c) Yes, $\lim_{n\to\infty} \frac{0.83}{1 + e^{-0.2n}} = 0.83$

SECTION 4.3 *(page 279)*

Prerequisite Review

1. $\frac{1}{2}e^x(2x^2 - 1)$ 2. $\frac{e^x(x + 1)}{x}$ 3. $e^x(x - e^x)$

4. $e^{-x}(e^{2x} - x)$ 5. $-\frac{6}{7x^3}$ 6. $6x - \frac{1}{6}$

7. $6(2x^2 - x + 6)$ 8. $\frac{t + 2}{2t^{3/2}}$

9. Relative maximum: $\left(-\frac{4\sqrt{3}}{3}, \frac{16\sqrt{3}}{9}\right)$

   Relative minimum: $\left(\frac{4\sqrt{3}}{3}, -\frac{16\sqrt{3}}{9}\right)$

10. Relative maximum: $(0, 5)$

    Relative minima: $(-1, 4), (1, 4)$

1. 3 3. -1 5. $4e^{4x}$ 7. $-2xe^{-x^2}$

9. $\frac{2}{x^3}e^{-1/x^2}$ 11. $e^{4x}(4x^2 + 2x + 4)$

13. $-\frac{6(e^x - e^{-x})}{(e^x + e^{-x})^4}$ 15. $xe^x + e^x + 4e^{-x}$

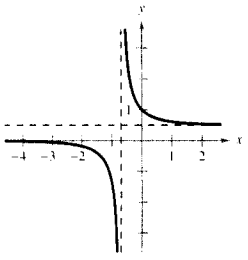
17. $y = -2x + 1$ 19. $y = \frac{4}{e^2}$ 21. $y = 24x + 8$

23. $\frac{dy}{dx} = \frac{1}{2}(-x - 1 - 2y)$ $\left(\text{Equivalently, } \frac{dy}{dx} = -\frac{1}{2}\right)$

25. $\frac{dy}{dx} = \frac{e^{-x}(x^2 - 2x) + y}{4y - x}$

27. $6(3e^{3x} + 2e^{-2x})$ 29. $5e^{-x} - 50e^{-5x}$

31.



No relative extrema

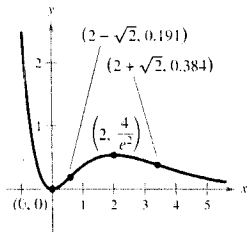
No points of inflection

Horizontal asymptote to the right: $y = \frac{1}{2}$

Horizontal asymptote to the left: $y = 0$

Vertical asymptote: $x \approx -0.693$

33.

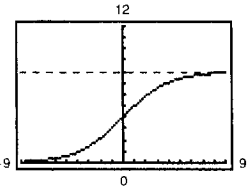


Relative minimum: $(0, 0)$

Relative maximum: $(2, 4e^{-2})$

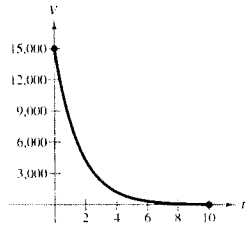
Points of inflection: $(2 - \sqrt{2}, 0.191), (2 + \sqrt{2}, 0.384)$

35.



Asymptotes: $y = 0, y = 8$

37. (a)

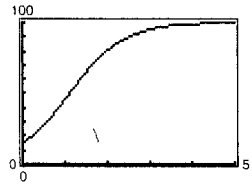


(b) $-\$5028.84$/year (c) $-\$406.89$/year

(d) $v = -1497.2t + 15{,}000$

(e) In the exponential function, the initial rate of depreciation is greater than in a linear model. The linear model has a constant rate of depreciation.

39. (a)



(b) 80.3%

(c) $x \approx 1.1$ or approximately 1100 egg masses

41. (a) \$433.31/year (b) \$890.22/year

(c) \$21,839.26/year

43. $t = 5$: 14.44

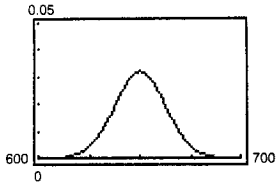
$t = 10$: 3.63

$t = 25$: 0.58

45. (a) $f(x) = \dfrac{1}{12.5\sqrt{2\pi}} e^{-(x-650)^2/2(12.5)^2}$

$= \dfrac{1}{12.5\sqrt{2\pi}} e^{-(x-650)^2/312.5}$

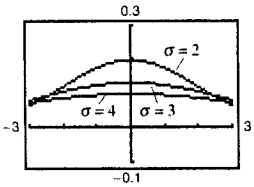
(b)



(c) $f'(x) = \dfrac{-4\sqrt{2}(x-650)e^{-2(x-650)^2/624}}{15625\sqrt{\pi}}$

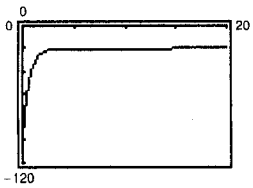
(d) Answers will vary.

47.



As σ increases, the graph becomes flatter.

49. (a) $\dfrac{dh}{dt} = -80e^{-1.6t} - 20$



(b) $-100, -36.15, -20.03, -20.00, -20.00$

(c) The values in (b) are rates of descent in feet per second. As time increases, the rate is approximately constant at -20 feet/second.

SECTION 4.4 *(page 287)*

Prerequisite Review

1. $\frac{1}{4}$ **2.** 64 **3.** 3^6 **4.** $\left(\frac{2}{3}\right)^3$ **5.** 1

6. $81e^4$ **7.** $\dfrac{e^3}{2}$ **8.** $\dfrac{125}{8e^3}$ **9.** $x > -4$

10. Any real number x **11.** $x < -1$ or $x > 1$

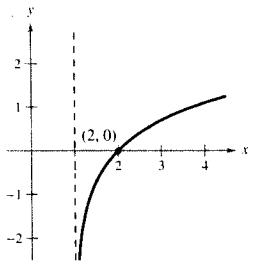
12. $x > 5$ **13.** \$3462.03 **14.** \$3374.65

1. $e^{0.6931\ldots} = 2$ **3.** $e^{-1.6094\ldots} = 0.2$ **5.** $\ln 1 = 0$

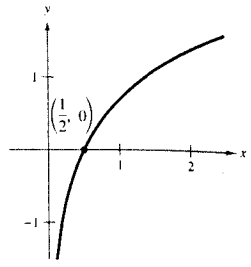
7. $\ln(0.0498\ldots) = -3$ **9.** c **10.** d

11. b **12.** a

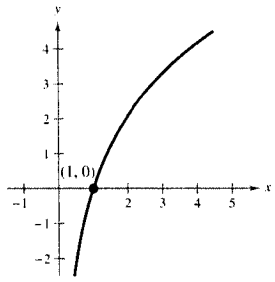
13.



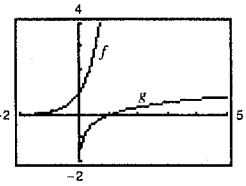
15.



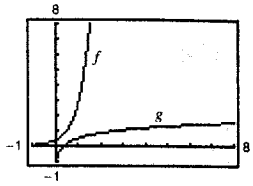
17.



19. Answers will vary.



21. Answers will vary.



23. x^2 **25.** $5x + 2$ **27.** $2x - 1$

29. (a) 1.7917 (b) 0.4055 (c) 4.3944 (d) 0.5493

31. $\ln 2 - \ln 3$ **33.** $\ln x + \ln y + \ln z$

35. $\frac{1}{2}\ln(x^2 + 1)$ **37.** $\ln z + 2\ln(z - 1)$

39. $\ln 3 + \ln x + \ln(x + 1) - 2\ln(2x + 1)$

41. $\ln\dfrac{x-2}{x+2}$ **43.** $\ln\dfrac{x^3y^2}{z^4}$ **45.** $\ln\left[\dfrac{x(x+3)}{x+4}\right]^3$

47. $\ln\left[\dfrac{x(x^2+1)}{x+1}\right]^{3/2}$ **49.** $\ln\dfrac{(x+1)^{1/3}}{(x-1)^{2/3}}$

51. $x = 4$ **53.** $x = 1$ **55.** $x = \ln 4 - 1 \approx 0.3863$

57. $t = \dfrac{\ln 7 - \ln 3}{-0.2} \approx -4.2365$

59. $x = \frac{1}{2}\left(1 + \ln\frac{3}{2}\right) \approx 0.7027$

61. $x = -100\ln\frac{3}{4} \approx 28.7682$

63. $x = \dfrac{\ln 15}{2\ln 5} \approx 0.8413$ **65.** $t = \dfrac{\ln 2}{\ln 1.07} \approx 10.2448$

67. $t = \dfrac{\ln 3}{12\ln[1 + (0.07/12)]} \approx 15.740$

69. (a) 14.21 years (b) 13.89 years

(c) 13.86 years (d) 13.86 years

71. $t \approx -12{,}194$ years

73. (a) $P($

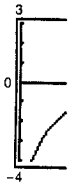
75. 9395

79. (a) 8C

81.

x
1
3
10
4

83.



85. False.

87. False.

SECTION

Prerec

1. 2 ln

3. ln x

5. ln 4

6. 3 ln

7. $-\frac{}{x}$

9. -12

1. 3 **3.**

11. $\dfrac{3}{x}(\ln x)^5$

17. $\dfrac{1}{x(x+1}$

23. $e^{-x}\left(\dfrac{1}{x}-\right.$

29. $\dfrac{1}{\ln 4}\ln x$

73. (a) $P(20) \approx 1{,}681{,}900$ (b) 2011

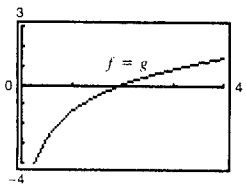
75. 9395 years **77.** 12,484 years

79. (a) 80 (b) 57.5 (c) 10 months

81.

x	y	$\frac{\ln x}{\ln y}$	$\ln \frac{x}{y}$	$\ln x - \ln y$
1	2	0	−0.6931	−0.6931
3	4	0.7925	−0.2877	−0.2877
10	5	1.4307	0.6931	0.6931
4	0.5	−2.0000	2.0794	2.0794

83.



85. False. $f(x) = \ln x$ is undefined for $x \leq 0$.

87. False. $f\left(\frac{x}{2}\right) = f(x) - f(2)$ **89.** False. $u = v^2$

SECTION 4.5 *(page 296)*

Prerequisite Review

1. $2\ln(x + 1)$ **2.** $\ln x + \ln(x + 1)$

3. $\ln x - \ln(x + 1)$ **4.** $3[\ln x - \ln(x - 3)]$

5. $\ln 4 + \ln x + \ln(x - 7) - 2\ln x$

6. $3\ln x + \ln(x + 1)$

7. $-\frac{y}{x + 2y}$ **8.** $\frac{3 - 2xy + y^2}{x(x - 2y)}$

9. $-12x + 2$ **10.** $-\frac{6}{x^4}$

1. 3 **3.** 2 **5.** $\frac{2}{x}$ **7.** $\frac{2x}{x^2 + 3}$ **9.** $\frac{2(x^3 - 1)}{x(x^3 - 4)}$

11. $\frac{3}{x}(\ln x)^5$ **13.** $x(1 + \ln x^2)$ **15.** $\frac{2x^2 - 1}{x(x^2 - 1)}$

17. $\frac{1}{x(x + 1)}$ **19.** $\frac{2}{3(x^2 - 1)}$ **21.** $-\frac{4}{x(4 + x^2)}$

23. $e^{-x}\left(\frac{1}{x} - \ln x\right)$ **25.** $\frac{e^x - e^{-x}}{e^x + e^{-x}}$ **27.** $e^{x(\ln 2)}$

29. $\frac{1}{\ln 4}\ln x$ **31.** 5.585 **33.** −0.631

35. −2.134 **37.** $(\ln 3)3^x$ **39.** $\frac{1}{x \ln 2}$

41. $(2 \ln 4)4^{2x-3}$ **43.** $\frac{2x + 6}{(x^2 + 6x)\ln 10}$

45. $2^x(1 + x \ln 2)$ **47.** $y = x - 1$

49. $y = \frac{1}{3 \ln 3}x - \frac{2}{9 \ln 3} + 2$ or $y = 0.303x + 1.798$

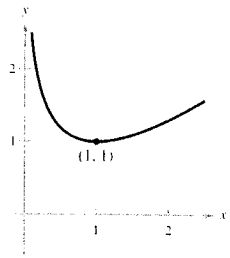
51. $\frac{2xy}{3 - 2y^2}$ **53.** $\frac{y(1 - 6x^2)}{1 + y}$ **55.** $\frac{1}{2x}$ **57.** $(\ln 5)^2 5^x$

59. $\frac{d\beta}{dI} = \frac{10}{(\ln 10)I}$; for $I = 10^{-4}$, the rate of change is about 43,429.4 decibels per watt per square centimeters.

61. $2, y = 2x - 1$ **63.** $-\frac{8}{5}, y = -\frac{8}{5}x - 4$

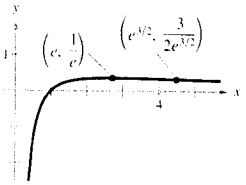
65. $\frac{1}{\ln 2}, y = \frac{1}{\ln 2}x - \frac{1}{\ln 2}$

67.



Relative minimum: (1, 1)

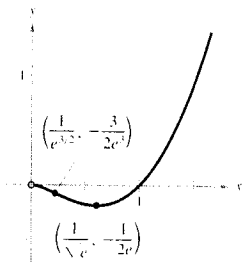
69.



Relative maximum: $\left(e, \frac{1}{e}\right)$

Point of inflection: $\left(e^{3/2}, \frac{3}{2e^{3/2}}\right)$

71.



Relative minimum: $\left(\frac{1}{\sqrt{e}}, -\frac{1}{2e}\right)$

Point of inflection: $\left(\frac{1}{e^{3/2}}, -\frac{3}{2e^3}\right)$

73. $-\frac{1}{p}, -\frac{1}{10}$ **75.** $-\frac{1000p}{(p^2+1)[\ln(p^2+1)]^2}, -4.65$

77. $p = 1000e^{-x}$

$\frac{dp}{dx} = -1000e^{-x}$

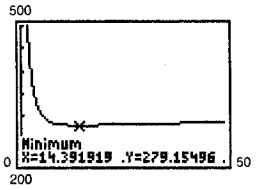
At $p = 10$, rate of change $= -10$.

$\frac{dp}{dx}$ and $\frac{dx}{dp}$ are reciprocals of each other.

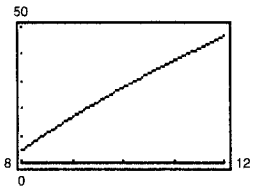
79. (a) $\overline{C} = \frac{500 + 300x - 300\ln x}{x}$

(b) Minimum of 279.15 at $e^{8/3}$

(c)



81. (a)



(b) \$27.56 billion (c) 10.33

83. Answers will vary.

SECTION 4.6 *(page 305)*

Prerequisite Review

1. $-\frac{1}{4}\ln 2$ **2.** $\frac{1}{5}\ln\frac{10}{3}$ **3.** $-\frac{\ln(25/16)}{0.01}$

4. $-\frac{\ln(11/16)}{0.02}$ **5.** $7.36e^{0.23t}$ **6.** $1.296e^{0.072t}$

7. $-33.6e^{-1.4t}$ **8.** $-0.025e^{-0.001t}$ **9.** 4

10. 12 **11.** $2x + 1$ **12.** $x^2 + 1$

1. $y = 2e^{0.1014t}$ **3.** $y = 4e^{-0.4159t}$

5. $y = 0.6687e^{0.4024t}$ **7.** $y = 10e^{2t}$, exponential growth

9. $y = 30e^{-4t}$, exponential decay

11. *Amount after 1000 years:* 6.48 grams

Amount after 10,000 years: 0.13 gram

13. *Initial quantity:* 6.73 grams

Amount after 1000 years: 5.96 grams

15. *Initial quantity:* 2.16 grams

Amount after 10,000 years: 1.62 grams

17. 68% **19.** 15,642 years

21. $k_1 = \frac{\ln 4}{12} \approx 0.1155$, so $y_1 = 5e^{0.1155t}$.

$k_2 = \frac{1}{6}$, so $y_2 = 5(2)^{t/6}$

Explanations will vary.

23. (a) 1350 (b) $\frac{5\ln 2}{\ln 3} \approx 3.15$ hours

(c) No. Answers will vary.

25. *Time to double:* 5.78 years

Amount after 10 years: \$3320.12

Amount after 25 years: \$20,085.54

27. *Annual rate:* 8.94%

Amount after 10 years: \$1833.67

Amount after 25 years: \$7009.86

29. *Annual rate:* 9.50%

Time to double: 7.30 years

Amount after 25 years: \$5375.51

31. (a) Answers will vary. (b) 6.17%

33.

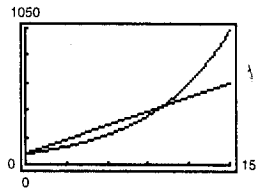
Number of compoundings/yr	4	12
Effective yield	5.095%	5.116%

Number of compoundings/yr	365	Continuous
Effective yield	5.127%	5.127%

35. Answers will vary.

37. (a) \$1034.08 million (b) \$628.25 million

(c)



$t = 0$ corresponds to 1993.

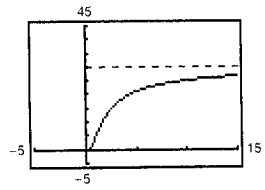
Answers will vary.

39. (a) $C = 30$

$k = \ln\left(\frac{1}{6}\right) \approx -1.7918$

(b) $30e^{-0.35836} \approx 20.9646$ or 20,965 units

(c)



41. About 36 days **43.** \$496.806

45. (a) $C = \frac{625}{64}$

$k = \frac{1}{100} \ln \frac{4}{5}$

(b) $x = 448$ units: $P = \$3.59$

47. 2046

REVIEW EXERCISES FOR CHAPTER 4

(page 312)

1. 8 **3.** 125 **5.** 1 **7.** $\frac{1}{6}$ **9.** 4

11. 16 **13.** $\frac{1}{2}$ **15.** $e\sqrt[3]{3}$

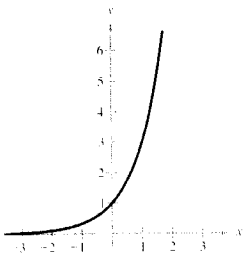
17. (a) 1995: $R(5) \approx \$524.04$ million

1998: $R(8) \approx \$636.58$ million

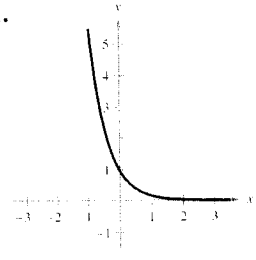
2001: $R(11) \approx \$773.30$ million

(b) Answers will vary.

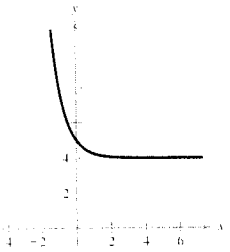
19.



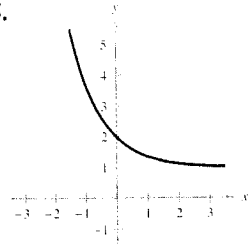
21.



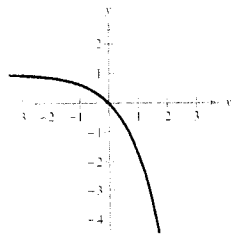
23.



25.



27.



29. \$7500

Explanations will vary.

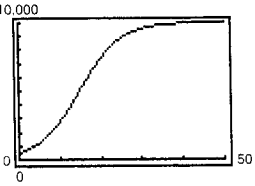
31. (a) $2e \approx 5.4366$ (b) $2e^{-1/2} \approx 1.2131$

(c) $2e^{9} \approx 16{,}206.168$

33. (a) $12e^{-3.4} \approx 0.4005$ (b) $12e^{-10} \approx 0.0005$

(c) $12e^{-20} \approx 2.4734 \times 10^{-8}$

35. (a)



(b) $P \approx 1049$ fish

(c) Yes. P approaches 10,000 fish as t approaches ∞.

(d) The population is increasing most rapidly at the inflection point, which occurs around $t = 15$ months.

37.

n	1	2	4	12
A	\$1216.65	\$1218.99	\$1220.19	\$1221.00

n	365	Continuous compounding
A	\$1221.39	\$1221.40

39. b **41.** (a) 6.14% (b) 6.17% **43.** \$9889.50

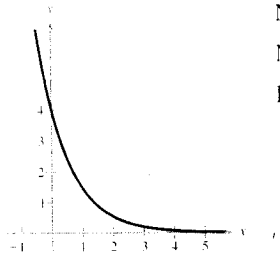
45. 1980: $P(0) = 24.196$ million

1995: $P(15) \approx 31.675$ million

2000: $P(20) = 32.700$ million

47. $8xe^{x^2}$ **49.** $\frac{1 - 2x}{e^{2x}}$ **51.** $4e^{2x}$ **53.** $\frac{-10e^{2x}}{(1 + e^{2x})^2}$

55.

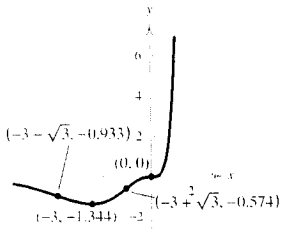


No relative extrema

No points of inflection

Horizontal asymptote: $y = 0$

57.



Relative minimum: $(-3, -1.344)$

Inflection points: $(0, 0)$, $(-3 + \sqrt{3}, -0.574)$, and $(-3 - \sqrt{3}, -0.933)$

Horizontal asymptote: $y = 0$