Transformations

To find a polynomial with specific roots
For example: Find a polynomials at \( x = -3, 0, 2 \)

\[ f(x) = -(x+3)(x-2) \]

6.3. Applications of Integrals
Integral represents area under curve

6.3.1 Areas
Areas between curves

Area between \( F(x) \) and \( g(x) \) is

\[ \int_{a}^{b} F(x) \, dx - \int_{a}^{b} g(x) \, dx = \int_{a}^{b} [F(x) - g(x)] \, dx \]
6.3.2 Cumulative Change

Find distance traveled

\[ \frac{ds}{dt} = 50 \text{ mph} \]

\[ \frac{ds}{dt} = 50 \]

\[ \int_{0}^{t} \frac{ds}{dt} \, dt = \int_{0}^{t} 50 \, dt \]

\[ s(t) - s(0) = 50t \]

\[ s(t) - s(0) = 50t \]

\[ s(t) \uparrow = 50(5) = 250 \text{ mi} \]

\[ \text{start from 0} \]

\[ a = -9.8 \text{ m/s}^2 = -32 \text{ ft/s}^2 \]

\[ a = \frac{dv}{dt} \]

\[ \int -9.8 \, dt = v(t) - v(0) \]

\[ -9.8t = v(t) - v(0) \]

\[ \int (v(t) = -9.8t + v_0) \, dt \]

\[ s(t) - s(0) = -\frac{1}{2} \cdot 9.8t^2 + v_0t \]

\[ s(t) = -\frac{1}{2} \cdot 0.8t^2 + v_0t + s_0. \]
How long does a ball take to fall from the top of a building that is 10 m tall?

\[ a = -9.8 \text{ m/s}^2 \]

\[ s_0 = \text{height of the building} \]

\[ u_0 = 0 \text{ m/s since ball is dropped} \]

\[ s(t) = -\frac{1}{2} \cdot 9.8 \cdot t^2 + u_0 t + s_0 \]

\[ s(t) = 0 = -\frac{1}{2} \cdot 9.8 t^2 + 10 \]

\[ t \approx 1.43 \text{ s} \]

What is the population of a colony of bacteria if the growth rate is \(0.5t\) where \(t\) is in hours and has an initial population of 500 after 7 hours?

\(N(t)\) is the population of the bacteria

\[
\int_{0}^{t} \frac{dN}{dt} = 0.5t \quad dt
\]

\[ N(t) - N(0) = \int_{0}^{t} 0.5t \quad dt \]

\[ = \frac{1}{2} \cdot 0.5t^2 \bigg|_{0}^{t} \]

\[ N(t) = 0.25t^2 + N(0) \]

\[ N(7) = 0.25(7)^2 + 500 \]

\[ = 12.25 + 500 \]

\[ = 512.25 \]

\[ \approx 512 \]
6.3.3. Average Values

If $f(x)$ is continuous and $y$ is between $f(a)$ and $f(b)$, there exists a $c$ between $a$ and $b$ such that $f(c) = y$.

$$A = \int_a^b f(x) \, dx$$
$$y(b-a) = \int_a^b f(x) \, dx$$
$$y = \frac{1}{b-a} \int_a^b f(x) \, dx$$

Find the average value of $y = 4 - x^2$ on the interval $[-2, 2]$.

$$y = \frac{1}{2-(-2)} \int_{-2}^{2} 4 - x^2 \, dx$$

$$= \frac{1}{4} \left( 4x - \frac{1}{3} x^3 \right) \bigg|_{-2}^{2}$$

$$= \frac{1}{4} \left( 8 - \frac{8}{3} + 8 - \frac{8}{3} \right) = \frac{8}{3}$$
Cumulative change example

The density of starfish along a beach is modeled by

\[ y = 3 + 4x \]

Find the total amount of starfish along the beach if the beach is 5 miles long.

\[
\int_0^x \frac{dN}{dx} = 3 + 4x \, dx \\
N(x) - N(0) = [3x + 2x^2]_0^x \\
N(x) = 3x + 2x^2 \\
N(5) = 3 \cdot 5 + 2 \cdot 5^2 \\
= 15 + 50 \\
= 65 \text{ starfish}
\]