Exercise 12: Let $\sigma : \mathbb{N} \to \mathbb{N}$ be a bijection, i.e. one-to-one and onto. Assume $(x_n)_{n \in \mathbb{N}}$ converges to $x \in \mathbb{R}$. Show that the sequence $(x_{\sigma(n)})_{n \in \mathbb{N}}$ also converges to the same $x \in \mathbb{R}$.

Exercise 13: Let $(x_n)$ be a bounded sequence a $S$ be the set of limit points of $(x_n)$, i.e.

$$S := \{ x \in \mathbb{R} : \text{there exists a subsequence } (x_{n_k}) \text{ s.t. } \lim_{k \to \infty} x_{n_k} = x \}$$

Show

$$\liminf_{n \to \infty} = \inf S.$$

*Hint: See lecture for proof lim sup.*

Exercise 14: (Caesaro revisited) Let $(x_n)$ be a convergent sequence. Let $(y_n)$ be the sequence given by

$$y_n = \frac{x_1 + \ldots + x_n}{n}$$

for all $n \in \mathbb{N}$. Show that

$$\limsup_{n \to \infty} y_n \leq \limsup_{n \to \infty} x_n.$$

Due: Monday October 21 in the lecture.