MAT 127A - Real Analysis - Fall 19

Exercise 1: Give an example/counterexample of the following

a) $B \subset \mathbb{R}$ with $\inf B \geq \sup B$.

b) A finite set which contains its infimum but not its supremum.

c) A bounded subset of $\mathbb{Q}$ which contains its supremum but not its infimum.

Exercise 2: Let $A \subset (0, \infty)$ and $B \subset [1, \infty)$. Show that the set $S := \frac{A}{B} := \left\{ \frac{a}{b} : a \in A, b \in B \right\}$ satisfies

$$\sup S = \frac{\sup A}{\inf B}$$ and $$\inf S = \frac{\inf A}{\sup B}.$$ Is this also true for any $B \subset \mathbb{R}$.

Exercise 3: Define the Fibonacci sequence recursively by $f_0 = f_1 = 1$ and $f_{n+1} = f_n + f_{n-1}$. Show that the greatest common divisor $\gcd(f_{n+1}, f_n) = 1$. 