Exercise 13: Suppose $\sum a_n$ and $\sum b_n$ converge absolutely. Prove $\sum a_n b_n$ converges absolutely.

Exercise 14: Does $\sum \frac{3^n n^2}{n!}$ converge?

Exercise 15: Let $a_k > 0$ be a positive sequence and $S_N = \sum_{k=1}^{N} a_k$. Suppose $\sum a_k$ diverges.

(a) Prove $\sum \frac{a_k}{1+a_k}$ diverges.

(b) Prove that:

$$\sum_{j=1}^{k} \frac{a_{N+j}}{S_{N+j}} \geq 1 - \frac{S_N}{S_{N+k}}$$

Deduce that $\sum \frac{a_k}{S_k}$ diverges as well.

Exercise 16: Use the Cauchy criterion to prove the alternating series test.

Remark: In the lecture we used the monotone convergence theorem.