Exercise 1: Let \( f(x) = \frac{2x - 3}{2x - 5} \). Does the equation \( f(x) = 0 \) has a solution between
   a) \( x = 0 \) and \( x = 1 \).
   b) \( x = 1 \) and \( x = 2 \).

Exercise 2: Compute the limits
   a) \( \lim_{x \to 0} \frac{\sqrt{x^4 + 1} - 1}{x} \).
   b) \( \lim_{x \to 0} \frac{\sqrt{x^2 + 9} - 3}{x^2} \).

Exercise 3: (Example of a differential equation: Logistic growth) Let \( N(t) \) be the population size at time \( t \). We call
\[
\frac{1}{N(t)} \frac{dN}{dt} = \frac{1}{N} \frac{dN}{dt}
\]
the per capita growth rate (we dropped the \( t \) dependence in the denominator in \( \frac{1}{N(t)} \frac{dN}{dt} \) to shorten notation, we also do this in the following). The quantity \( \frac{1}{N} \frac{dN}{dt} \) gives you the change of the population at time \( t \) divided by the size of the population at time \( t \), therefore per capita growth rate. We assume now that the capita growth rate satisfies
\[
\frac{1}{N} \frac{dN}{dt} = \left( 1 - \frac{N}{K} \right)
\]
with initial condition \( N(0) > 0 \), i.e. the population is not initially 0 and some \( K > 0 \), which we call the carrying capacity.
   a) Find all initial conditions such that \( \frac{dN}{dt} = 0 \). We call such points equilibria of the equation.
   b) Show that
\[
N(t) = \frac{KN_0}{N_0 + (K - N_0)e^{-t}}
\]
solves equation (1) with initial condition \( N(0) = N_0 \).
(You can use \( \frac{d}{dx} e^{-t} = -e^{-t} \))
c) Compute
\[ \lim_{t \to \infty} N(t). \]
d) Plot \( N(t) \) using \( R \) between \( t = 0 \) and \( t = 10 \) for initial conditions \( N_1 = 1, N_2 = 10 \) and \( N_3 = 15 \) and \( K = 20. \)

(Remark:
1. The bigger the carrying capacity \( K \) in the equation the higher the limiting population for \( t \) going to infinity.
2. Compare the exercise to the discrete logistic equation done in class.)

Exercise 4: Let \( c > 0 \) and
\[ f(x) = \frac{c}{x}, \quad x > 0. \quad (2) \]
a) Find the tangent line to the graph of the function at the point \( (x_0, f(x_0)) \).
Compute the point where the tangent line intersects the \( x \)-axis and conclude that this point is independent of \( c \).
b) Find the normal line to the graph of \( f \) in the point \( (1, 1) \). (The normal line is perpendicular to the tangent line, if the tangent line has the equation \( y = ax + b \) then the slope of the normal line is \( -a^{-1} \))