Problems

Hand in your best work on each of the problems below. Two of the problems in this section will be randomly selected to be graded.

1. You visit an island in which every inhabitant is either a liar or a truth teller - liars always lie and truth tellers always tell the truth. You meet two people, Ug and Bug. Ug says “Either I am a liar, or Bug is a truth-teller.” Is Ug a liar or a truth teller? What about Bug? Why?

2. Section 1.1, Exercise 1 (a), (b), (e), (i): Which of the following are propositions? Give the truth value of each proposition.
   (a) What time is dinner?
   (b) It is not the case that \( \pi \) is not a rational number.
   (e) Either \( \pi \) is rational and 17 is a prime, or 7 < 13 and 81 is a perfect square.
   (i) It is not the case that 39 is prime, or that 64 is a power of 2.

3. Section 1.1, Exercise 3 (a), (b), (d), (g), (k): Make a truth table for each of the following propositional forms. (Recall: \( \lor \) means ‘or’, \( \land \) means ‘and’, \( \neg \) means ‘not’.)
   (a) \( P \land \neg P \)
   (b) \( P \lor \neg P \)
   (d) \( P \land (Q \lor \neg Q) \)
   (g) \( (P \lor \neg Q) \land R \)
   (k) \( P \land P \)

4. Section 1.2, Exercise 1 and 2 for parts (b), (c), (e), (g): Identify the antecedent and consequent for each of the following conditional sentences. (That is, write the statements in the form \( P \implies Q \) for some statements \( P \) and \( Q \).) Also write the converse and contrapositive of each of these sentences.
   (b) If the moon is made of cheese, then 8 is an irrational number.
   (c) \( b \) divides 3 only if \( b \) divides 9.
   (e) A sequence \( a \) is bounded whenever \( a \) is convergent.
   (g) \( 1 + 2 = 3 \) is necessary for \( 1 + 1 = 2 \).

5. Section 1.2, Exercise 7 (a), (d), (e): Make truth tables for these propositional forms.
   (a) \( P \implies (Q \land P) \)
   (d) \( (P \lor Q) \implies (P \land Q) \)
   (e) \( (P \land Q) \lor (Q \land R) \implies (P \lor R) \).

Recommended extra practice problems

1. Section 1.1, Exercise 1 (j).
2. Section 1.1, Exercise 9.
3. Section 1.2, Exercise 12
4. Play through all the Circle Dot levels at [http://proveitmath.org/toyproofs/CircleDot.html](http://proveitmath.org/toyproofs/CircleDot.html)
Bonus Problem

In the Circle Dot formal proof system described at [http://proveitmath.org/toyproofs/CircleDot.html](http://proveitmath.org/toyproofs/CircleDot.html), which statements are provable, and which are not? Why?