You have 50 minutes to complete this exam. No books, references, cell phones, or communication of any sort is allowed during the exam. Write your answers in the space given below each problem, and if you need more room for a problem, you can go to the back of a page.

The table below is not to be filled in by you; it is for grading use only.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Out of:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1</td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>Problem 2</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>Problem 3</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Problem 4</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Problem 5</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Total:</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>
1. Short answer (25 points):

(a) (8 points) Fill in the truth table below for the statement: \( P \iff (\neg Q \land R) \).

<table>
<thead>
<tr>
<th></th>
<th>Q</th>
<th>R</th>
<th>\neg Q</th>
<th>\neg Q \land R</th>
<th>P \iff (\neg Q \land R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

(b) (4 points per part) Identify the antecedent and consequent of each of the following implications, and write the converse and contrapositive of each statement.

i. If a number is prime, then it’s not composite.

   Antecedent: If a number is prime.
   Consequent: It’s not composite.
   Converse: If it’s not composite, then a number is prime.
   Contrapositive: If a number is not prime, then it’s composite.

ii. I’ll go to the play only if Bob will be there.

   Antecedent: I’ll go to the play.
   Consequent: Bob will be there.
   Converse: If Bob will be there, then I’ll go to the play.
   Contrapositive: If I don’t go to the play, then Bob won’t be there.

iii. The condition \(|x| > 3\) is necessary for \(x^2 > 9\).

   Antecedent: \(|x| > 3\).
   Consequent: \(x^2 > 9\).
   Converse: If \(x^2 > 9\), then \(|x| > 3\).
   Contrapositive: If \(x^2 \leq 9\), then \(|x| \leq 3\).
(c) (1 point each) Translate the following into quantified statements using the quantifiers ∀, ∃, ∃!. All variables in the statement must be quantified. The parenthetical after the statement is the universe in which all variables are typed.

i. Everyone is confused. (All people)

ii. Exactly one person is confused. (All people)

iii. Only Bob is confused. (All people)

iv. For every real number $x$, there is a real number $y$ for which $x^2 = y^3$. (Real numbers)

v. A natural number $n$ is divisible by 3 if and only if there exists a natural number $k$ for which $3k = n$. (Natural numbers)
2. (15 points) Fill in the input line numbers for each of the reasons in the following formal proof in the Propositional Logic axiom system. Blanks (□) are given for each of the numbers that should be filled in. Each line is one point.

**Theorem 1.** \(\neg(P \Rightarrow Q) \Rightarrow (P \text{ and } \neg Q)\)

**Proof.**

1. Assume \(\neg(P \Rightarrow Q)\) -
2. Assume \(\neg P\) -
3. Assume \(P\) -
4. Assume \(\neg Q\) -
5. \(P\) copy; □
6. \(\neg P\) copy; □
7. \(\rightarrow\) \(\leftarrow\) \(\rightarrow\) \(\leftarrow\) +; □ □ □
8. \(\leftarrow\) -
9. \(Q\) ¬−; □ □ □
10. \(\leftarrow\) -
11. \(P \Rightarrow Q\) ⇒ +; □ □ □
12. \(\neg(P \Rightarrow Q)\) copy; □
13. \(\rightarrow\) \(\leftarrow\) \(\rightarrow\) \(\leftarrow\) +; □ □
14. \(\leftarrow\) -
15. \(P\) ¬−; □ □ □
16. Assume \(Q\) -
17. Assume \(P\) -
18. \(Q\) copy; □
19. \(\leftarrow\) -
20. \(P \Rightarrow Q\) ⇒ +; □ □ □
21. \(\neg(P \Rightarrow Q)\) copy; □
22. \(\rightarrow\) \(\leftarrow\) \(\rightarrow\) \(\leftarrow\) +; □ □
23. \(\leftarrow\) -
24. \(\neg Q\) ¬+; □ □ □
25. \(P \text{ and } (\neg Q)\) and +; □ □
26. \(\leftarrow\) -
27. \(\neg(P \Rightarrow Q) \Rightarrow (P \text{ and } \neg Q)\) ⇒ +; □ □ □
3. (20 points) Use induction to prove that for all natural numbers \( n \geq 2, \)
\[
2^2 + 2^3 + 2^4 + \ldots + 2^n = 2^{n+1} - 4.
\]
4. (20 points) Prove that $x(x + 1)$ is even for all natural numbers $x$.

(Your proof should be written in an informal/essay style, and take the definition of $m$ being even to be that there exists an integer $k$ for which $2k = m$. You may also use the fact that every natural number is either even or odd, and that $m$ being odd means that there exists an integer $j$ for which $2j + 1 = m$. )
5. (20 points) Prove that for any sets $A$, $B$, and $C$, if $A \subseteq B$ then $A - C \subseteq B - C$.

(You may use facts and tautologies from propositional logic freely without proof, but you must be fully rigorous in your use of definitions for subsets and set operations.)