Problems

Hand in your best work on each of the problems below. Two of the problems in this section will be randomly selected to be graded.

For the first three problems below, give a formal two-column proof of the statement, using the statements and rules of inference of Propositional Logic. You may NOT use any of the tautologies discussed in class or in the textbook; every reason in your proof must be one of the rules of inference on Page 7 of the Supplementary Lecture Notes (https://www.math.ucdavis.edu/~mgillespie/math108spring19/LectureNotes.pdf). Also draw its truth table (worth 1 point) and verify that it is a tautology.

1. (5 points) 
   \[(P \lor Q) \land \neg P \Rightarrow Q\]

2. (5 points) 
   \[(P \Rightarrow Q) \land \neg Q \Rightarrow \neg P\]

3. (5 points) 
   \[\neg (\neg P) \iff P\]

4. (4 points) Give an informal proof that \(\sqrt{5}\) is irrational. (You may use any facts you know from basic algebra and arithmetic, and you may use the theorem that if a prime \(p\) divides the product \(a \cdot b\) then either \(p\) divides \(a\) or \(p\) divides \(b\).)

5. (5 points) Section 1.5, exercise 12 parts (a), (c), (d), (e), (f).

Extra practice problems

These are not to be handed in. They are only for your own practice, and are recommended study problems for the midterm or final.

1. Section 1.4, exercises 5, 7, 9.

2. Section 1.5, exercises 4, 6, 7, 9.

3. Prove the following tautologies, each of which show a logical equivalence of two statements:
   
   - \(P \lor Q \iff Q \lor P\) (Commutativity of or)
   - \(P \land Q \iff Q \land P\) (Commutativity of and)
   - \((P \lor Q) \lor R \iff P \lor (Q \lor R)\) (Associativity of and)
   - \((P \land Q) \land R \iff P \land (Q \land R)\) (Associativity of or)
   - \(P \lor (Q \land R) \iff (P \lor Q) \land (P \lor R)\) (Distributive Law 1)
   - \(P \land (Q \lor R) \iff (P \land Q) \lor (P \land R)\) (Distributive Law 2)
   - \(\neg (P \lor Q) \iff (\neg P \land \neg Q)\) (DeMorgan’s Law 1)
   - \(\neg (P \land Q) \iff (\neg P \lor \neg Q)\) (DeMorgan’s Law 2)
   - \((P \Rightarrow Q) \iff (\neg Q \Rightarrow \neg P)\) (Contrapositive)

Bonus Problem

Give a formal proof of the statement \(P \lor \neg P\).

(Remember: Bonus problems are optional. If you hand them in, your work will be scored and counted towards your Bonus Point tally, which has nothing to do with your grade.)