Mathematics for Decision Making: An Introduction

Lecture 1

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Class Schedule, Office Hours, etc.

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Class schedule: Tuesdays / Thursdays 3:10–4:30

Office hours: Tuesdays / Thursdays 4:45–6:00 (negotiable)

Exams, homework, grading: see syllabus

Introduction

Some questions for you all:

- What's your major?
- What math (and other) lectures have you already taken?
- How much computer programming have you done? Which language(s)?
- What are your career goals?
- What's the one thing you'd like to learn in this class?

About “Mathematics for Decision Making”

Mathematics for Decision Making means to:

- analyze business processes (such as production, logistics, finances)
- create mathematical models for the processes
- use mathematical software to solve them
- make good (or even optimal) decisions on how to change these processes to make them better
- have these decisions implemented

Another name for this field: **Operations Research**
Professional Societies:

- INFORMS, see also `http://www.scienceofbetter.org/`
- Mathematical Programming Society

Recommended literature

No requirement to buy any of these, but very useful reading.

- **Combinatorial Optimization**, by William J. Cook, William H. Cunningham, William R. Pulleyblank, Alexander Schrijver – most of my lecture is based on this book; should be available in the bookstore
- **Combinatorial Optimization – Algorithms and Complexity**, by Christos H. Papadimitriou and Kenneth Steiglitz – a classic and must-have, with a very cheap paperback edition ($\leq$ \$20), pretty dense, though
- **Optimization in Operations Research**, by Ronald L. Rardin – a gentle introduction, covers many aspects of optimization, but is short on proofs
- **Linear Programming: Foundations and Extensions**, by Robert J. Vanderbei – the text of MAT 168, introduces all topics as consequences of linear optimization (which we do not cover in this class)

Case Study: MULTITRANS

MULTITRANS is a fictional bus company that is run by a student association in an unnamed college town with partial funding from the city. (It started out in 1972 with a fleet of historic triple-deck buses from Moscow, but has added more modern buses since.)

During most of the day, though, all scheduled buses are completely empty.

A scientific study was conducted in 2008 to find out why this is the case. The result of the study was:

> *[...] 85% of the representative sample of 1256 potential users of MULTITRANS replied that "I would use it regularly instead of using my bike or car, but the MULTITRANS schedule [is suboptimal]." [...]*

MULTITRANS hires you as a consultant to help them improve the schedule.

What are the next steps you need to do?

Let's start with a simpler example, though

(From the Rardin book.)

The Notip Table Company sells two models of its patented five-leg tables. The basic version uses a wood top, requires 0.6 hours to assemble, and sells for a profit of $200. The deluxe model takes 1.5 hours to assemble (because of its glass top), and sells for a profit of $350. Over the next week the company has 300 legs, 50 wood tops, 35 glass tops, and 63 hours of assembly available. Notip wishes to determine a maximum profit production plan assuming that everything produced can be sold.

Mathematical Optimization Problem, abstractly

Mathematical Optimization Problem, abstract definition

$$\begin{aligned} \max \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{x} \in F \end{aligned}$$

Here

- “max” means “maximize!”
- f is a function: the *objective function*
- “s.t.” stands for “subject to (constraints)” or “such that”
- F is a set: the set of all *feasible solutions*
- $\mathbf{x}$ is one feasible solution

X can be finite or infinite.

Which optimization problems are easier to solve?

Standard Form of Deterministic, Finite-Dimensional Mathematical Optimization Problems

$$
\begin{aligned}
\max \quad & f(\mathbf{x}) \\
\text{s.t.} \quad & g_1(\mathbf{x}) \le 0 \\
& \vdots \\
& g_m(\mathbf{x}) \le 0 \\
& \mathbf{x} = (x_1, \ldots, x_n) \in X
\end{aligned}
$$

where $f, g_1, \ldots, g_m \colon \mathbf{R}^n \to \mathbf{R}$ are functions and $X = \mathbf{R}^n$, or $X = \mathbf{Z}^n$, or $X = \mathbf{R}^{n_1} \times \mathbf{Z}^{n_2}$.

We classify optimization problems according to the properties of the functions f and g_i and the space X.

Homework

For Thursday Jan 8 (not graded):

- Get a class account at:

  `http://www.math.ucdavis.edu/comp/class-accts`

  (This class is MAT-180-1)
- Run SCIP, SoPLEX, and ZIMPL on the department computers and find out their version numbers.
  The executable programs live in the directory `~mkoeppe/mat180/`.
- Browse the ZIMPL manual.