Mathematics for Decision Making: An Introduction

Lecture 3

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January 14, 2009

The Traveling Salesperson Problem

Traveling Salesperson Problem (TSP)

A traveling salesperson needs to visit n cities, starting from city 1, visiting each other city exactly once, and returning to city 1. Find the shortest possible such tour.

"Distances" between the cities can be measured in different ways:

- If cities are considered as points $(x_i, y_i) \in \mathbf{R}^2$, a useful distance between cities could be the Euclidean distance

$$d(i,j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}.$$

- Other distances that make sense are: Number of miles on the shortest road connection between cities i and j, airfares in US dollars, ...
  Some of these distance functions d have some of these properties:
  - Satisfies triangle inequality (Metric TSP):

$$d(i,j) \leq d(i,k) + d(k,j)$$

  - Symmetry (Symmetric vs. Asymmetric TSP):

$$d(i,j) = d(j,i)$$

Some comments on the TSP

The TSP is one of the most famous combinatorial optimization problems.

- It appears in a wide array of applications.
  - Various route planning problems
  - Printed circuit board drilling
  - Application in genome sequencing
- It is notoriously hard to solve.
  - A symmetric TSP has $(n-1)!/2$ possible tours, so brute force is not feasible.
  - Simple, intuitive heuristics such as the "visit-nearest-neighbor algorithm" fail spectacularly
  - No "efficient" algorithm is known for it today, and it belongs to a huge class (*NP-hard*) of other, seemingly unrelated, difficult problems that we could suddenly solve efficiently, once an efficient algorithm for TSP became available.
- We'll find out how well our black-box optimization solver works for this model

Excellent web resource, by Prof. William Cook (Georgia Tech):

`http://www.tsp.gatech.edu/`

- Concorde (state-of-the-art software for TSP), world records on the TSP, applications, and illustrations.

Modeling the TSP as a standard optimization problem

Variables?

Constraints (inequalities)?